Time Value of Money Formulas

Simple Interest

The simple interest, I, on P dollars at a rate of interest r per year for t years is

$$I = Prt.$$

Example: A company borrows $5,000 at 6% for two months. How much interest will they have to pay?

$P = \$5{,}000$; $r = 6\%$ per year; $t = 2/12$ years. Note that the interest rate is annual. To find the interest rate per month you must divide by 12 and then multiply by two because the loan is for two months.

$$I = \$5{,}000 \times .06 \times 2 \div 12 = \$50$$

Future Maturity Value for Simple Interest

The future value or maturity value, FV, of P dollars for t years at a rate of interest r per year is

$$FV = P + I = P + Prt = P(1+rt).$$

Example: A company borrows $5,000 at 6% for two months. How much will they have to pay at the end of the loan?

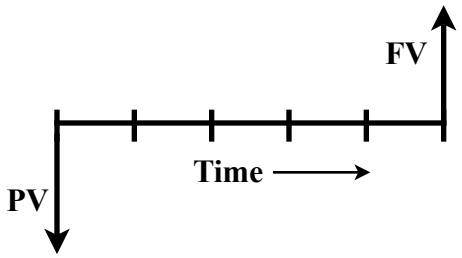
$P = \$5{,}000$; $r = 6\%$ per year; $t = 2/12$ years. Note that the interest rate is annual. To find the interest rate per month you must divide by 12 and then multiply by two because the loan is for two months.

$$FV = \$5{,}000\,(1+(.06 \times 2 \div 12)) = \$5{,}000\,(1+.01) = \$5{,}050$$

*Simple interest formulas are very easy to solve with any calculator. You will find the formulas for compound interest more challenging, and most students are prone to make mistakes when entering them into the calculator. If you have a TI 83/84 you can easily solve compound interest problems without the need to use the formulas. Press **[APPS]**, select **"1:Finance"**, then select **"TVM Solver"**. In the pages that follow I give instructions on how to use TVM Solver to solve common finance problems.*

Future Value for Compound Interest

If *PV* dollars are deposited for *n* compounding periods at a rate of interest *i* per period, the compound amount (or future value) *FV* is



$$FV = PV(1+i)^n.$$

If the annual interest rate *r* is compounded *m* times per year and the number of years is *t*, then ***i = r/m*** and ***n = mt.***

Example: A company invests $12,000 for 5 years at 4% compounded monthly. How much will they have at the end of 5 years?

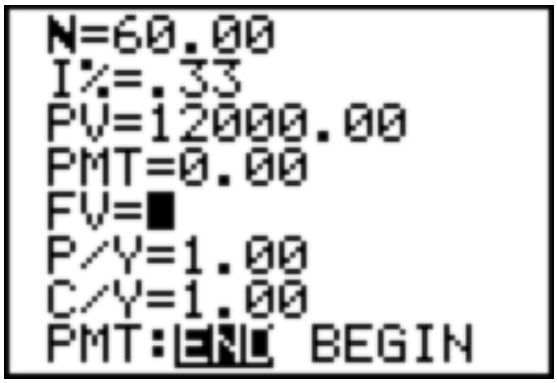
PV =$12,000; i =.04/12 (monthly interest rate); n =5 x 12 (months)

$FV = \$12{,}000\,(1+(.04/12))^{60} = \$12{,}000\,(1.221) = \$14{,}651.96$

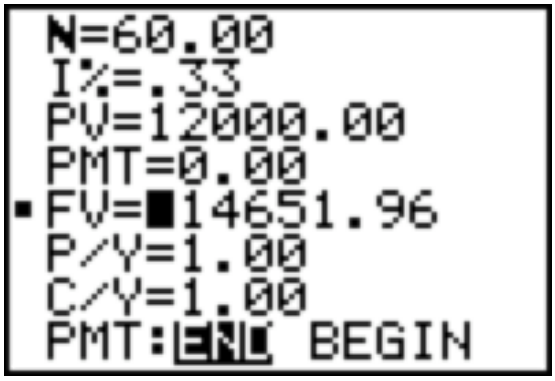
Using TVM Solver in the TI 83/84

Enter the following values and leave the cursor blinking on FV as shown.

Note that you can do calculations as you enter the values in each line. To enter N you can type 5x12 and to enter I% you can type 4/12. The calculator will perform the operation and display the results to the right. Also keep in mind that it is better to let the calculator do these operations, as 4% divided by 12 is .333333333333333... The calculator keeps all the decimal points, though it displays only the number of decimals you selected in [MODE], [FLOAT].

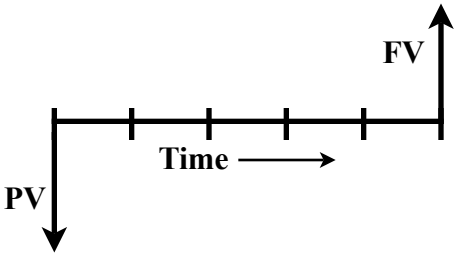


With the cursor blinking on FV, press the green [ALPHA] key, followed by [ENTER]. You will get the FV shown to the right.



Present Value for Compound Interest

The present value of *FV* dollars compounded at an interest rate *i* per period for *n* periods is



$$PV = FV/(1+i)^n \quad or \quad PV = FV(1+i)^{-n}$$

If the annual interest rate *r* is compounded *m* times per year and the number of years is *t*, then ***i = r/m*** and ***n = mt.***

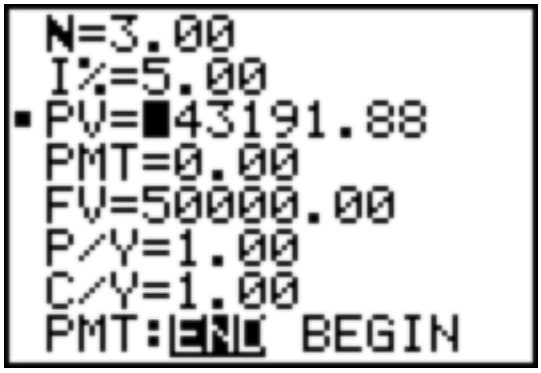
Example: A company needs to buy a $50,000 machine three years from now. How much will they have to invest today at 5% compounded annually to accumulate that amount?

FV =$50,000; i =5% (annual interest rate); n =3 (number of years)
$PV = \$50{,}000\ (1+.05)^{-5} = \$43{,}191.88$

Using TVM Solver in the TI 83/84

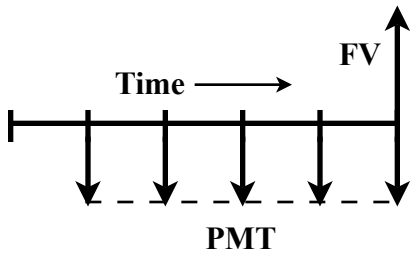
Enter the following values and leave the cursor blinking on PV as shown.

With the cursor blinking on PV, press the green [ALPHA] key, followed by [ENTER]. You will get the PV shown to the right.



Future Value of an Ordinary Annuity

The future value *FV* of an ordinary annuity is given by



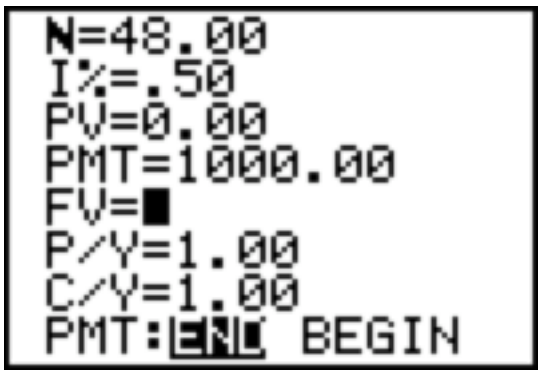
$$FV = PMT\left(\frac{(1+i)^n - 1}{i}\right)$$

If the annual interest rate *r* is compounded *m* times per year and the number of years is *t*, then ***i = r/m*** and ***n = mt.***

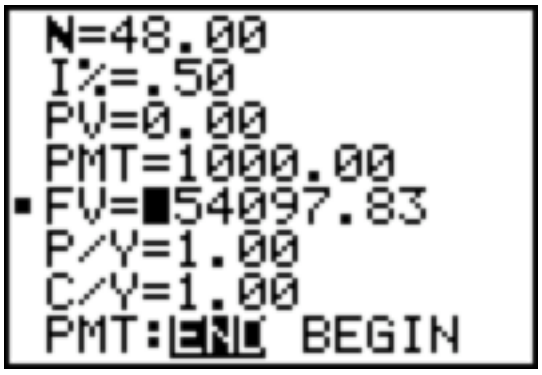
Example: A company deposits $1,000 at the end of every month for the next four years in an account earning 6% compounded monthly. How much will the company have accumulated at the end of the four years.

PMT =$1,000; i =6%/12 (monthly interest rate); n =4x12 (number of months)
FV = $1,000 [(1+(.06/12))^{48} –1] / (.06/12) = $54,097.83

Using TVM Solver in the TI 83/84
Enter the following values and leave the cursor blinking on FV as shown.
Note that you can do calculations as you enter the values in each line. To enter N you can type 4x12 and to enter I% you can type 6/12. The calculator will perform the operation and display the result shown to the right. Also keep in mind that it is better to let the calculator do these operations, as it keeps all the decimals, though it displays only the number of decimals you selected in [MODE], [FLOAT].

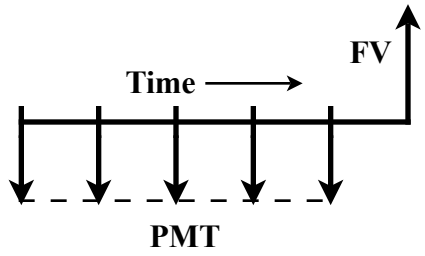


With the cursor blinking on FV, press the green [ALPHA] key, followed by [ENTER]. You will get the FV shown to the right.



Future Value of an Annuity Due

The future value *FV* of an annuity due is given by



$$FV = PMT\left(\frac{(1+i)^{n+1} - 1}{i}\right) - PMT$$

If the annual interest rate *r* is compounded *m* times per year and the number of years is *t*, then ***i = r/m*** and ***n = mt.***

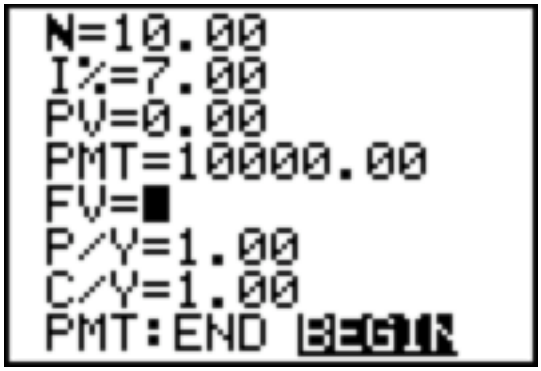
Note that in an annuity due payments are made at the beginning of the period and not at the end of the period like in an ordinary annuity.

Example: A company deposits $10,000 at the beginning of each year for ten years in an account paying 7%. How much will they have saved after the ten years.

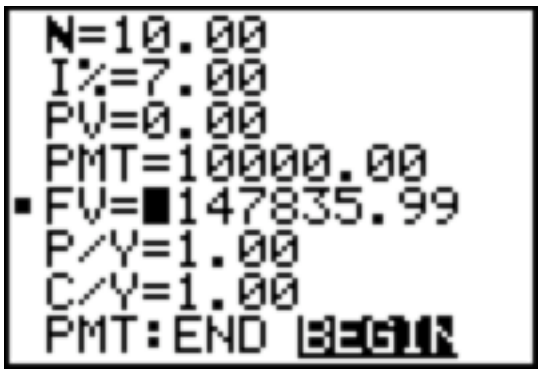
PMT =$10,000; i =7% (annual interest rate); n =10 (number of years)

FV = $10,000 [((1+.07)$^{10+1}$ – 1) / .07] – $20,000 = $147,835.99

Using TVM Solver in the TI 83/84
Enter the following values and leave the cursor blinking on FV as shown.
Note that for an annuity due you must set the last line to BEGIN. This tells the calculator that payments are to be made at the beginning and not at the end of the period.

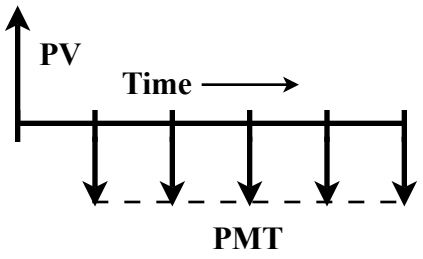


With the cursor blinking on FV, press the green [ALPHA] key, followed by [ENTER]. You will get the FV shown to the right.



Present Value of an Ordinary Annuity

The present value *PV* of an annuity of *n* payments of *PMT* dollars each at the end of consecutive interest periods with interest compounded at a rate of interest *i* per period is



$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

If the annual interest rate *r* is compounded *m* times per year and the number of years is *t*, then ***i = r/m*** and ***n = mt.***

Example: A company sells equipment to a customer and accepts 36 equal monthly payments of $2,516.84 in return. Typical interest rates for this type of loan are 10%. What was the sales price of the equipment?

PMT =$2,516.84; i =.10/12 (monthly interest rate); n =36 (months)

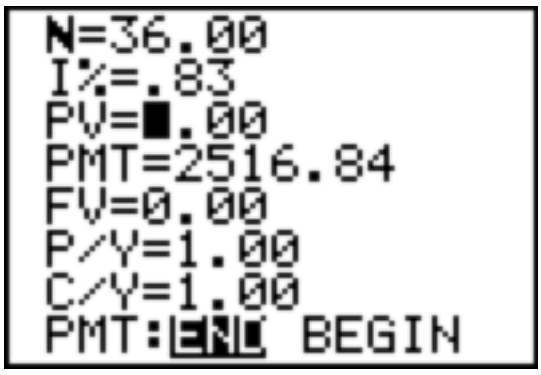
$PV = \$2{,}516.84\,[1-(1+(.10/12))^{-36}] / (.10/12)$

PV =$77,999.98

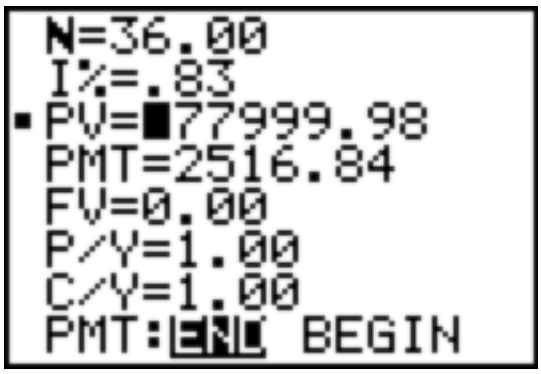
Using TVM Solver in the TI 83/84

Enter the following values and leave the cursor blinking on PV as shown.

Note that you can do calculations as you enter the values in each line. To enter I% you can type 10/12. The calculator will perform the operation and display the result shown to the right. Also keep in mind that it is better to let the calculator do these operations, as it keeps all the decimals, though it displays only the number of decimals you selected in [MODE], [FLOAT].

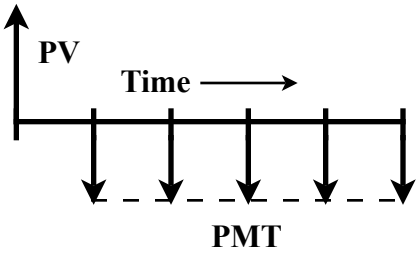


With the cursor blinking on PV, press the green [ALPHA] key, followed by [ENTER]. You will get the PV shown to the right.



Amortization Payments

A loan of P dollars at interest rate i per period may be amortized in n equal periodic payments of A dollars made at the end of each period, where



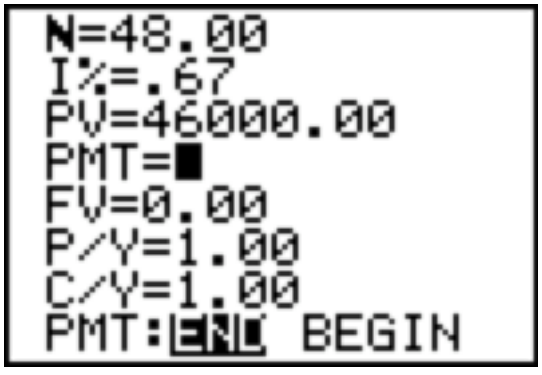
$$PMT = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = \frac{Pi}{1-(1+i)^{-n}}$$

If the annual interest rate r is compounded m times per year and the number of years is t, then $\boldsymbol{i = r/m}$ and $\boldsymbol{n = mt}$.

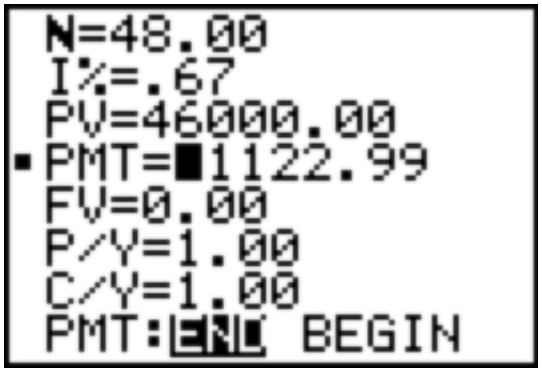
Example: You buy a $46,000 car and agree to pay it off in 48 equal monthly payments at 8%. What is the monthly payment?

PV =$46,000; i =.08/12 (monthly interest rate); n =48 (months)
PMT = [$46,000 (.08/12)] / [1 –(1+(.08/12))$^{-48}$]
PMT =$1,122.99

Using TVM Solver in the TI 83/84
Enter the following values and leave the cursor blinking on PMT as shown.
Note that you can do calculations as you enter the values in each line. To enter I% you can type 8/12. The calculator will perform the operation and display the result shown. Also, keep in mind that it is better to let the calculator do these operations, as it keeps all the decimals, though it displays only the number of decimals you selected in [MODE], [FLOAT].



With the cursor blinking on PMT, press the green [ALPHA] key, followed by [ENTER]. You will get the PMT as shown to the right.



Additional Formulas

Simple Discount Notes

Some simple interest loans deduct the interest in advance.

FV = the maturity value of the loan
P = the proceeds, or what the borrower actually receives
I = the interest on the loan. Also called the discount, D

$$P = FV - I = FV - FVrt = FV(1 - rt)$$

Example: You borrow $10,000 from the bank at a 12% discount rate for 6 months. How much will you get from the bank and what will be you total payment?

FV =$10,000 (the maturity value, what you have to pay at the end of six months)
i =.12/12 (monthly interest rate); n =6 (months)
I = $10,000 x .12/12 x 6 = $600 (the interest or discount)
P = $9,400 (the proceeds or what you will receive)

Continuous Compounding

The compound amount *FV* for a deposit of *P* dollars at interest rate *r* per year compounded continuously for *t* years is given by

$$FV = P e^{rt}$$

Example: You invest $10,000 at an annual rate of 6% compounded continuously for 10 years. What will be the future value of your investment?

P = $10,000; r = .06; t = 10 years
$FV = \$10{,}000 \times e^{(.06 \times 10)} = \$18{,}221.19$

Effective Interest Rate

The effective rate corresponding to a stated rate of interest *r* compounded m times per year is

$$r_e = (1 + r/m)^m - 1$$

Example: Calculate the effective rate for a 6% loan compounded monthly.

$$r_e = (1 + .06/12)^{12} - 1 = 6.17\%$$