A rectangular matrix is in **echelon form** if the following conditions apply:

1. All nonzero rows are above any rows of zeros.
2. Each leading entry of a row is in a column to the right of the leading entry above it.
3. All entries in a column below a leading entry are zeros.

A matrix is in **reduced echelon form** if it satisfies the additional properties:

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only nonzero entry in its column.

Note that any nonzero matrix may be row reduced (transformed by row operations) into more than one matrix in echelon form. However, reduced echelon form is unique. That is, each matrix is row-equivalent to exactly one reduced echelon matrix.

A **pivot position** in a matrix A is a location in matrix A that corresponds to a leading 1 in the reduced echelon form of A. A **pivot column** is a column that contains a pivot position.

Example 1 Reduce the matrix A to echelon form and locate the pivot columns of A.

$$A = \begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}.$$

Interchange row 1 and row 4.

Create zeros below the pivot.

Create zeros below the new pivot. Note that row 3 is now a row of zeros.

We interchange row 3 and row 4 to get a leading entry.

Columns 1, 2 and 4 are the pivot columns.

This algorithm leads to an explicit description of the solution set of a linear system. Consider the reduced echelon form and the corresponding linear system.

$$\begin{bmatrix} 1 & 0 & -5 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} x_1 \quad - 5x_3 &= 1 \\ x_2 + x_3 &= 4 \\ 0 &= 0 \end{aligned}$$

The variables x_1 and x_2 are the **basic variables** (or **leading variables**). The variable x_3 is called a **free variable**. The solution set can be described explicitly by solving the reduced system:

$$\begin{aligned} x_1 &= 1 + 5x_3 \\ x_2 &= 4 - x_3 \\ x_3 &\text{ is free} \end{aligned}$$

Each choice of x_3 produces a different solution of the system and every solution of the system is determined by a choice of x_3.The description above is called the **general solution**.