Example 1 Solve the linear system below. Write the augmented matrix for each step of the process.

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9 \end{aligned} \qquad \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix}$$

Keep x_1 in the first equation and eliminate it from the other equations. Add 4 times equation (row) 1 to equation (row) 3. Replace equation (row) 3 with the result.

Multiply equation (row) 2 by ½ in order to obtain 1 as the coefficient of x_2.

Keep x_2 in the second equation and eliminate it from the third equation. Add 3 times equation (row) 2 to equation (row) 3. Replace equation (row) 3 with the result.

Keep x_3 in the third equation and eliminate it from the other equations.

Finally, keep x_2 in the second equation and eliminate it from the first equation.

Operations on equations in a linear system correspond to operations on the rows of the augmented matrix. These operations are called **elementary row operations**. There are three elementary row operations:

1. (**Replacement**) Replace one row with the sum of itself and a multiple of another row.
2. (**Interchange**) Interchange two rows.
3. (**Scaling**) Multiply all entries of a row by a nonzero constant.

Two matrices are **row equivalent** if there is a sequence of elementary row operations that transforms one matrix into the other.

If the augmented matrices of two linear systems are row equivalent, then the two systems have the same solution set.

Example 2 Solve the system of equations below using elementary row operations on the augmented matrix.

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 15 \\ -2x_1 + 2x_2 - 2x_3 &= -14 \\ -4x_1 + 4x_2 + x_3 &= -8 \end{aligned}$$

Example 3 Determine if the linear system below is consistent using elementary row operations on the augmented matrix.

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 5x_1 - 8x_2 + 7x_3 &= 1 \end{aligned}$$