MATH 252: Calculus II

Name: ________________

Review I

Definition of Limit

We say that $\lim_{x \to c} f(x) = L$ if and only if for every $\varepsilon > 0$, there is a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

1. Explain this definition in your own words.

2. If $f(x) = x^2$, $c = 2$ and $\varepsilon = 0.1$, then find δ.

3. Illustrate the definition of a limit.

Definition of the Derivative

The derivative of f at $x = c$, denoted $f'(c)$, is defined as $f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$. If the limit exists, we say that f is differentiable at $x = c$.

4. In terms of the graph of f, how do we interpret $f'(c)$?

5. If $f(t)$ represents the temperature of a pie t minutes after being placed in a hot oven, how do we interpret the statement $f'(20) = 2$? What are the units of $f'(20)$?

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Review I

Derivative Formulas

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}a^x = a^x \ln a \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\tan x = \sec^2 x$$

Derivative Rules

Product Rule: $\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$

Chain Rule: $\frac{d}{dx}f\left(g(x)\right) = f'\left(g(x)\right)\cdot g'(x)$

In Problems 6-10, find the derivative of the function. Simplify.

6. $y = e^{f(x)}$

7. $f(x) = \frac{(3x-4)^3}{(5-2x)^4}$

8. $y = \cos\sqrt{x^2 + 2x - 1}$

9. $f(x) = xe^{\tan x}$

10. $y = \ln\left(f(x)\right)$

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Review II

The Definite Integral

If f is continuous on the interval $[a,b]$, then the definite integral of f from a to b, denoted $\int_a^b f(x)\,dx$, tells us the signed area bounded by the graph of f, the x-axis, and the lines $x=a$ and $x=b$. That is, the bounded area below the x-axis is counted negatively.

Estimate the value of the definite integral from a graph, a table of values or a formula.

The Fundamental Theorem of Calculus

If f is continuous on $[a,b]$, and $f(x)=F'(x)$, then $\int_a^b f(x)\,dx = F(b)-F(a)$. That is, the definite integral of a rate of change gives the total change.

In Problems 1 and 2, consider the following.

A car traveling 80 ft/sec brakes to a stop in 8 seconds. Its velocity is recorded every 2 seconds and is given in the following table.

t (seconds)	0	2	4	6	8
$v(t)$ (ft/sec)	80	52	28	10	0

1. Write a definite integral that gives the distance traveled by the car during the 8 seconds. Estimate the distance traveled by the car during the 8 seconds.

2. How often must the velocity of the car be recorded to estimate the distance traveled to within 20 feet?

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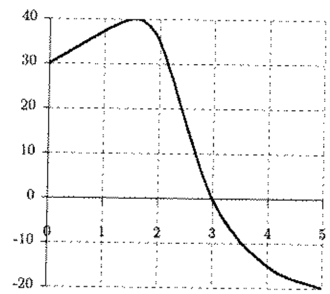
Review II

In Problems 3 and 4, consider the following.

The graph below gives your velocity (in kph) during a trip starting from home. Positive velocities take you away from home and negative velocities take you toward home.



3. Write a definite integral that represents your distance from home after 5 hours. How far are you from home after 5 hours?

4. When are you farthest from home? How far from home are you at this time?

In Problems 5 and 6, estimate the definite integral to the nearest thousandth.

5. $\displaystyle\int_0^2 \sqrt{1+x^3}\ dx$

6. $\displaystyle\frac{1}{\sqrt{2\pi}}\int_0^1 e^{-x^2/2}\ dx$

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Review III

Differentiation on the TI-83 Plus

1. Let $f(x) = e^{-x^2}$. Find $f'(1)$, accurate to the nearest thousandth.

There are two options for calculating derivatives at a point on the TI-83 Plus. The first is by using the nDeriv function on the home screen. The second involves the graph of the function. Start by pressing [MATH] to bring up the menu in Figure 1. Scroll down to highlight 8:nDeriv or press 8 to bring this function to the home screen as shown in Figure 2. Now enter the function followed by X followed by 1 (the point at which you are evaluating the derivative), separated by commas, as shown in Figure 3. Finally, press [ENTER] to calculate the result (See Figure 4.). Thus, $f'(1) \approx -0.7358$.

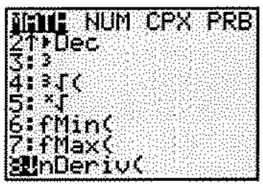

Figure 1

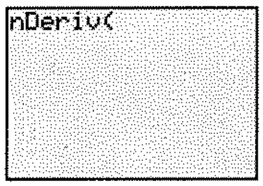

Figure 2

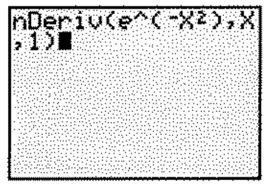

Figure 3

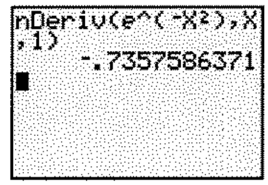

Figure 4

Syntax: nDeriv(function, X, point)

We can also evaluate the derivative at a point using the graph of the function. First, produce the graph of the function, as shown in Figure 5. (This example uses the window $[-2.35, 2.35] \times [-1.55, 1.55]$.) Press [2nd] [TRACE] to bring up the CALCULATE menu shown in Figure 6. Scroll down to 6:dy/dx or press 6. You will be returned to the graph and prompted for an X-value. Enter 1 (the point at which you are evaluating the derivative) as shown in Figure 7 and press [ENTER]. The point on the graph is highlighted and the derivative is shown at the bottom of the screen (See Figure 8). Thus, $f'(1) \approx -0.7358$.

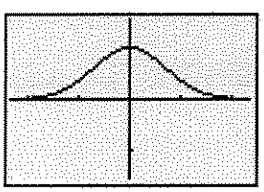
Figure 5

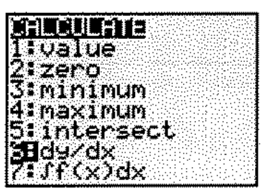

Figure 6

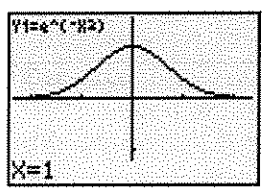

Figure 7

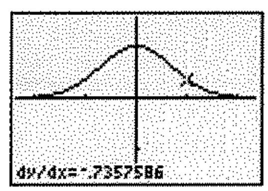

Figure 8

You can also find the equation of the tangent line to calculate the derivative at a point. First, produce the graph of the function, as shown in Figure 9. Press [2nd] [PRGM] to bring up the DRAW menu shown in Figure 10. Scroll down to 5:Tangent or press 5. You will be returned to the graph and prompted for an X-value. Enter 1 (The point at which you are evaluating the derivative.) as shown in Figure 11 and press [ENTER]. The line tangent to the graph of the function is drawn and the equation for this line is shown at the bottom of the screen (See Figure 12.). The slope of the tangent line is -0.7358. Thus, $f'(1) \approx -0.7358$.

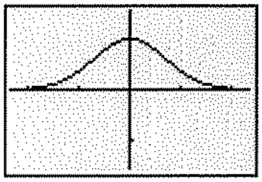

Figure 9

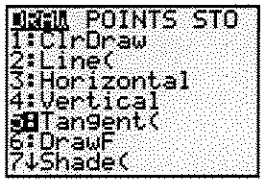

Figure 10

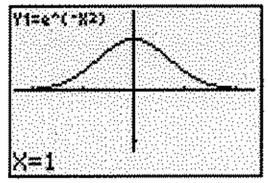

Figure 11

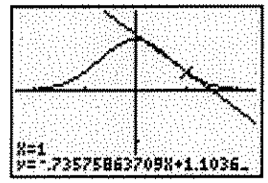

Figure 12

Definite Integrals on the TI-83 Plus

2. Let $f(x) = e^{-x^2}$. Find $\int_0^1 f(t)\,dt$, accurate to the nearest thousandth.

There are two options for calculating definite integrals on the TI-83 Plus. The first is by using the fnInt function on the home screen. The second involves the graph of the function. Start by pressing MATH to bring up the menu in Figure 13. Scroll down to highlight 9:fnInt or press 9 to bring this function to the home screen as shown in Figure 14. Now enter the function followed by X followed by 0 (the lower limit) followed by 1 (the upper limit), separated by commas, as shown in Figure 15. Finally, press ENTER to calculate the result (See Figure 16.). Thus, $\int_0^1 e^{-x^2} dx \approx 0.7468$.

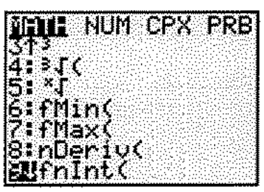

Figure 13

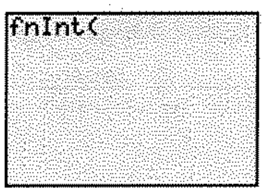

Figure 14

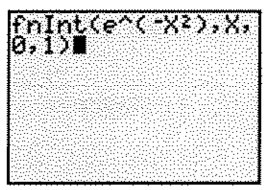

Figure 15

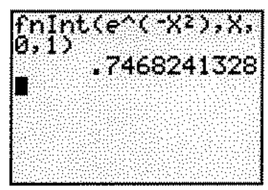

Figure 16

Syntax: fnInt(function, X, lower limit, upper limit)

We can also evaluate the definite integral using the graph of the function. First, produce the graph of the function, as shown in Figure 5. Press 2nd TRACE to bring up the CALCULATE menu shown in Figure 17. Scroll down to 7:∫f(x)dx or press 7. You will be returned to the graph and prompted for lower limit. Enter 0 (the lower limit) as shown in Figure 18 and press ENTER. You will then be prompted for an upper limit. Enter 1 (the upper limit) as shown in Figure 19 and press ENTER. The area is shaded and the value of the definite integral is shown at the bottom of the screen (See Figure 20.). Thus,

$$\int_0^1 e^{-x^2} dx \approx 0.7468.$$

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Review III

Figure 17

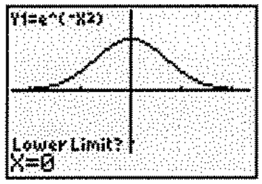

Figure 18

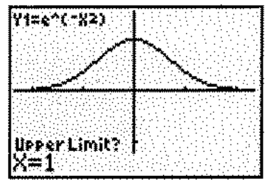

Figure 19

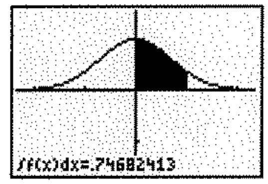

Figure 20

Plotting the Derivative of a Function on the TI-83 Plus

3. Let $f(x) = e^{-x^2}$. Graph f and f' on the same set of axes.

First, enter the function $f(x) = e^{-x^2}$ as Y_1. For Y_2, press MATH, then 8, then ENTER to bring the nDeriv function to the Y= screen as shown in Figure 21. Rather than entering the function again, press VARS and then scroll right to bring up the Y-VARS menu shown in Figure 22. Select 1:Function to access the FUNCTION menu and select 1:Y_1 from that menu. Enter X followed by X (the point at which you are evaluating the derivative) separated by commas as shown in Figure 23. Finally, press GRAPH to display the graph of the function and its derivative shown in Figure 24.

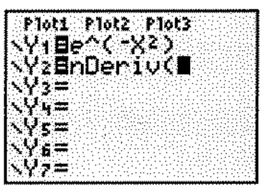

Figure 21

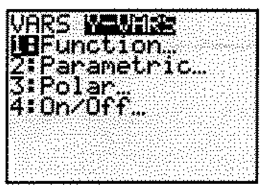

Figure 22

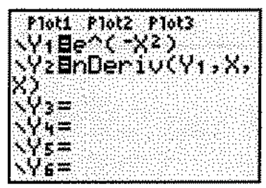

Figure 23

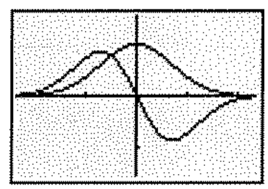

Figure 24