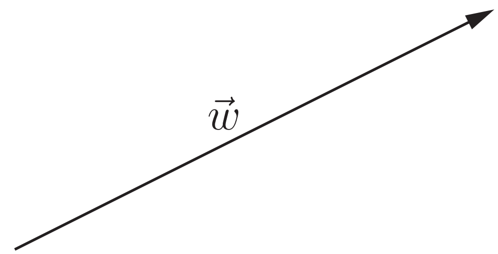
Notes

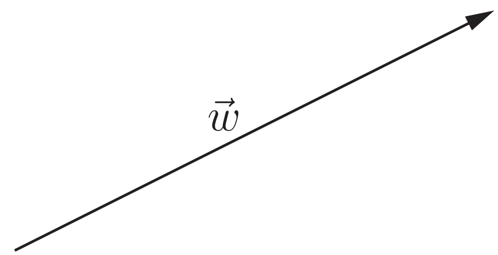
Projection

Suppose you have a vector, $\vec{w}$



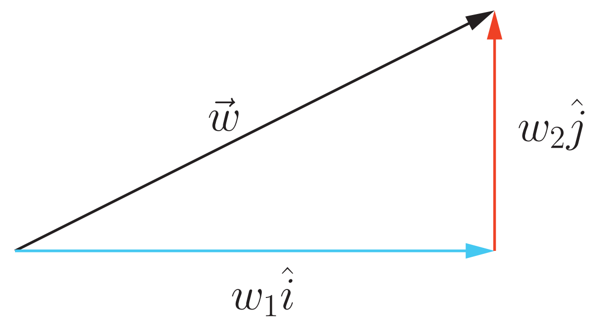
Projection

Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.



Projection

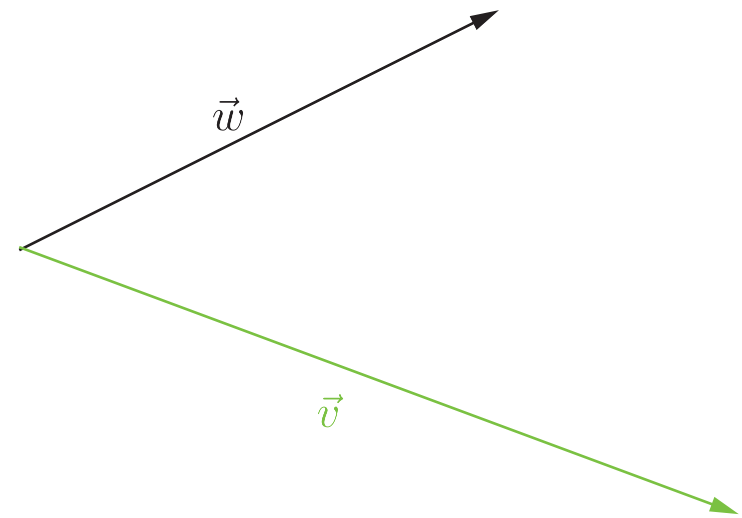
Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
Typically we use horizontal and vertical vectors.



$$\vec{w} = w_1\hat{i} + w_2\hat{j}$$

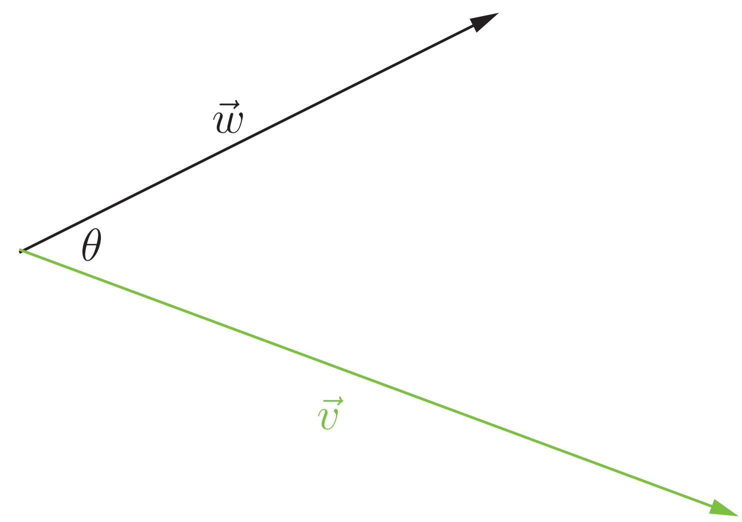
Projection

Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
But there's no rule that says you can't break it down into components based on an arbitrary direction – like vector $\vec{v}$.



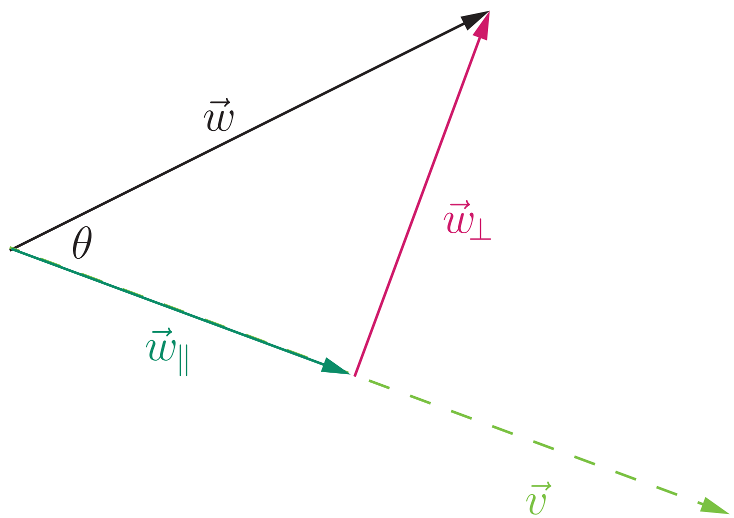
Projection

Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
But there's no rule that says you can't break it down into components based on an arbitrary direction – like vector $\vec{v}$.
Note θ, the angle between $\vec{w}$ and $\vec{v}$.



Projection

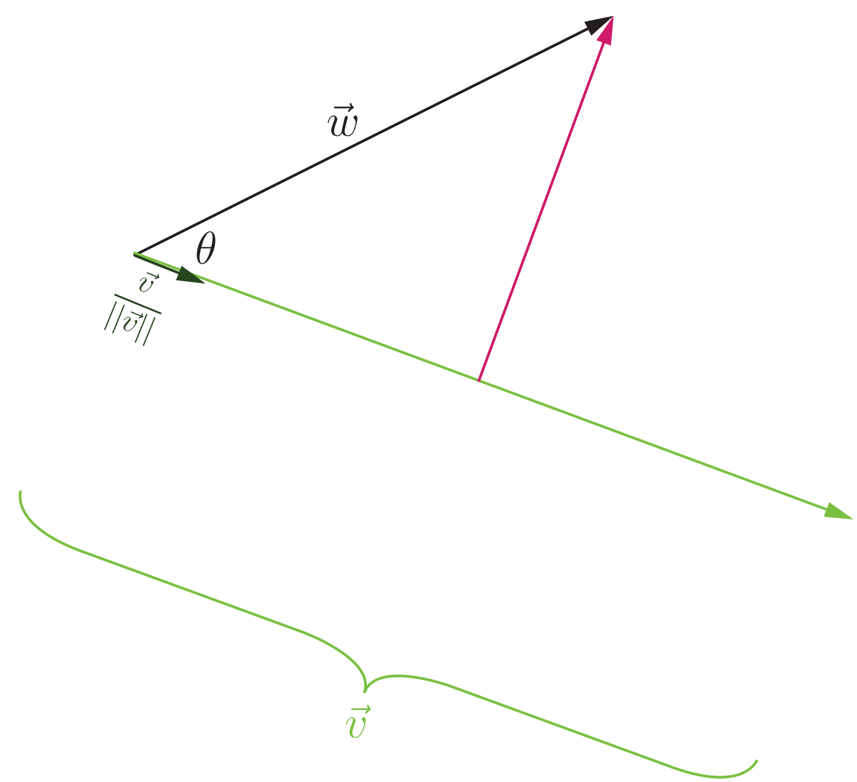
Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
But there's no rule that says you can't break it down into components based on an arbitrary direction – like vector $\vec{v}$.
Then we have a component of $\vec{w}$ parallel to the direction of $\vec{v}$ and a component of $\vec{w}$ perpendicular to the direction of $\vec{v}$.



$$\vec{w} = \vec{w}_\parallel + \vec{w}_\perp$$

Projection

Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
To find the component of $\vec{w}$ in the direction of $\vec{v}$ (the projection of $\vec{w}$) we begin with the unit vector of $\vec{v}$, $\dfrac{\vec{v}}{||\vec{v}||}$ to give us direction.



Projection

Suppose you have a vector, $\vec{w}$,
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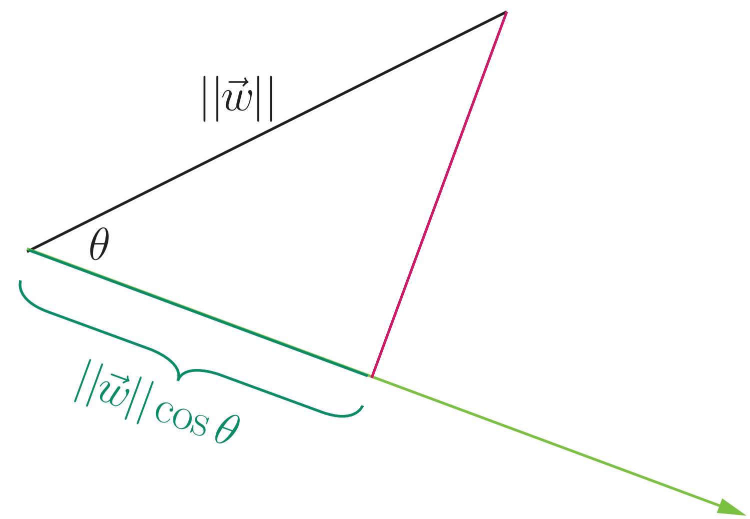
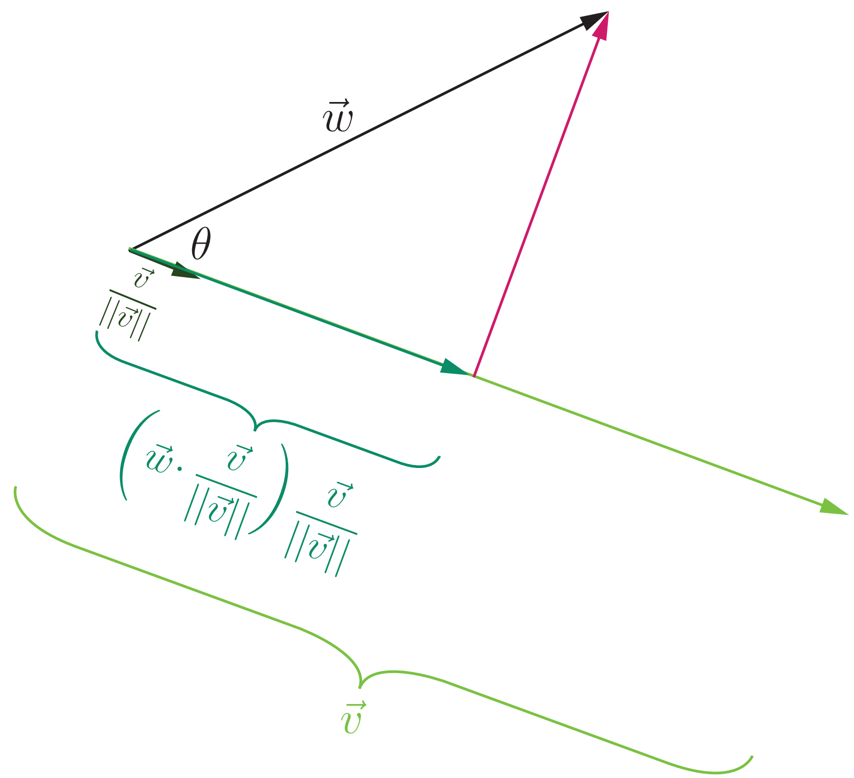
To find the component of $\vec{w}$ in the direction of $\vec{v}$ (the projection of $\vec{w}$) we begin with the unit vector of $\vec{v}$, $\dfrac{\vec{v}}{||\vec{v}||}$ to give us direction.
Then since the magnitude of the vector in the direction of $\vec{v}$ is given by $||\vec{w}|| \cos\theta$,



Projection

Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
To find the component of $\vec{w}$ in the direction of $\vec{v}$ (the projection of $\vec{w}$) we begin with the unit vector of $\vec{v}$, $\dfrac{\vec{v}}{||\vec{v}||}$ to give us direction.
Then since the magnitude of the vector in the direction of $\vec{v}$ is given by $||\vec{w}||\cos\theta = \vec{w}\cdot\dfrac{\vec{v}}{||\vec{v}||}$, we have a component of $\vec{w}$ parallel to $\vec{v}$:

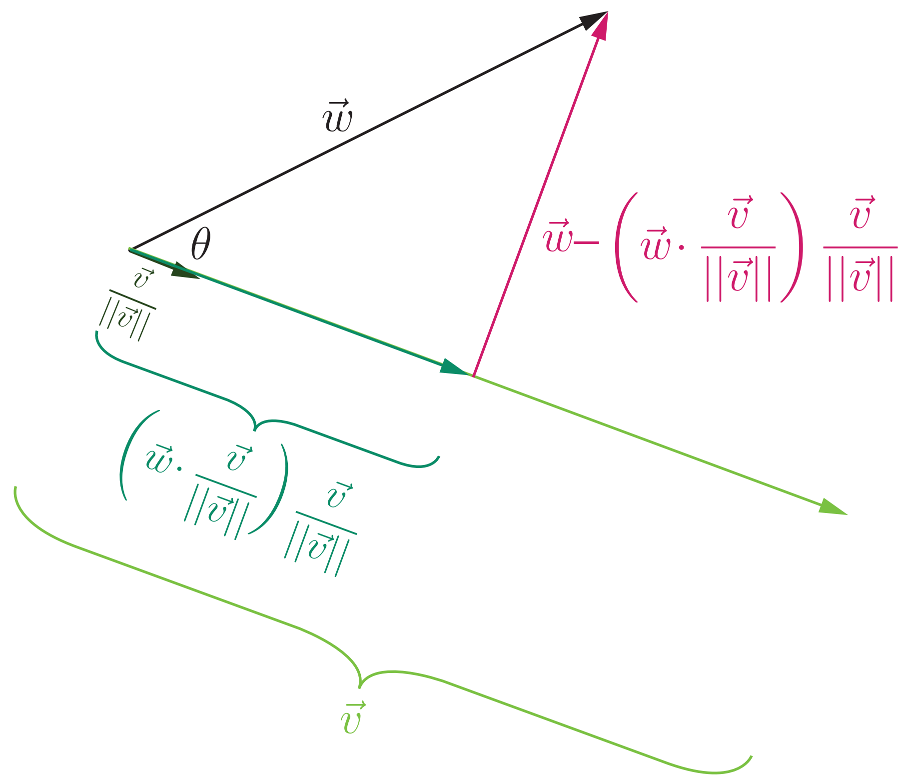
$$\vec{w}_{\parallel} = \left(\vec{w}\cdot\frac{\vec{v}}{||\vec{v}||}\right)\frac{\vec{v}}{||\vec{v}||}$$



Projection

Suppose you have a vector, $\vec{w}$,
and you want to break it down into two orthogonal (perpendicular) components.
To find the component of $\vec{w}$ in the direction of $\vec{v}$ (the projection of $\vec{w}$) we begin with the unit vector of $\vec{v}$, $\dfrac{\vec{v}}{||\vec{v}||}$ to give us direction.
It follows the vector orthogonal (perpendicular) to $\vec{w}_{\parallel}$ is given by

$$\vec{w}_{\perp} = \vec{w} - \left(\vec{w} \cdot \frac{\vec{v}}{||\vec{v}||}\right) \frac{\vec{v}}{||\vec{v}||}$$



Example

Decompose $\vec{w} = 2\hat{i} - 3\hat{j}$ into components parallel and perpendicular to the vector $\vec{v} = 3\hat{i} - \hat{j}$.

Solution:

We know $\vec{w}_{||} = \left(\vec{w} \cdot \frac{\vec{v}}{||\vec{v}||}\right) \frac{\vec{v}}{||\vec{v}||} = \left(\frac{\vec{w} \cdot \vec{v}}{||\vec{v}||^2}\right) \vec{v}$

Then $\vec{w} \cdot \vec{v} = 6 + 3 = 9$

and $||\vec{v}||^2 = \left(\sqrt{3^2 + (-1)^2}\right)^2 = 10$

So $\vec{w}_{||} = \frac{9}{10}(3\hat{i} - \hat{j}) = 2.7\hat{i} - 0.9\hat{j}$

It follows that since $\vec{w}_{\perp} = \vec{w} - \vec{w}_{||}$ we have $\vec{w}_{\perp} = 2\hat{i} - 3\hat{j} - (2.7\hat{i} - 0.9\hat{j}) = -0.7\hat{i} - 2.1\hat{j}$ $\blacksquare$