Math 252

Work and Density Name:________________________

YOU MAY USE A CALCULATOR TO COMPUTE SOLUTIONS BUT SHOW YOUR SET-UPS.

Show all relevant work!

(1) Find the center of mass (relative to the base) of a hemisphere of radius r and uniform density ρ.

Ans: $\overline{x} = 0, \overline{y} = \frac{3}{8}r$

(2) Find the center of mass of the solid of revolution formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^3$ about the y–axis. Assume uniform density.

Ans: $\overline{x} = 0, \overline{y} = \frac{25}{48}$

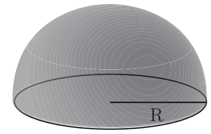
(3) Find the center of mass (both coordinates) of a right-triangular plate of uniform thickness and density whose height is 20cm and width is 14cm.

Ans: $\overline{x} = \frac{14}{3}, \overline{y} = \frac{20}{3}$

(4) A bucket that weighs 4lb. and a rope of negligible weight are used to draw water from a well that is 80 feet deep. The bucket starts with 40lb. of water and is pulled up at a rate of 2 ft/sec. Unfortunately, there is a hole in the bucket and water leaks out at a rate of 0.2lb/sec. Find the work done in pulling the bucket to the top of the well.

Ans: 3200 ft·lb.

(5) Determine the work done by gravity in emptying a hemispherical tank of water. Assume the radius of the tank is 8m and the weight density of water is 10,000 Newtons/m^3.



Ans: $\frac{5632000}{3}\pi \approx 58978166$ Newtons.

(6) Newton's Law of Gravitation states that two bodies with masses m_1 and m_2 attract each other with a force

$$F = G\frac{m_1 m_2}{r^2}$$

where r is the distance between the bodies and G is the gravitational constant ($G \approx 6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$). If one of the bodies is fixed, find the work needed to move the other from a distance $r = a$ to a distance $r = b$.

Ans: $Gm_1m_2\left(\frac{1}{a} - \frac{1}{b}\right)$

(7) Use Newton's Law of Gravitation to compute the work required to launch a 1000 kg satellite vertically into an orbit 1000 km high. You may assume that Earth's mass is 5.98×10^{24}kg and is concentrated at its center. Take the radius of Earth to be 6400 km.

Ans: 8.42×10^{12} Newtons