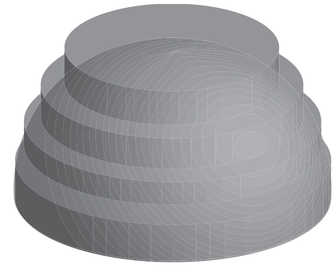
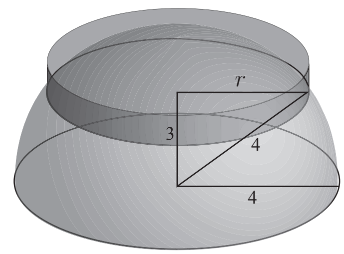
Use slices to approximate the volume of a hemisphere whose radius is 4cm. Taking four horizontal slices (of thickness 1cm) we have cylindrical discs that we have to sum up. The only missing part is their radius. For the LHS the bottom disc has a radius of 4cm. What about the next three?



Consider the top disc which is located 3cm from the bottom. If we draw the radius ofthe disc, r, the height, and the radius of the hemisphere, then we see a familiar right triangle. For the top disc $r = \sqrt{16-9} = \sqrt{5}$. Similarly, for the other two discs we have $r = \sqrt{16-1}$ and $r = \sqrt{16-4}$. It follows that:



LHS $= \pi(4)^2 + \pi(\sqrt{15})^2 + \pi(\sqrt{12})^2 + \pi(\sqrt{5})^2 = \sum_{i=0}^{3} \pi \left(\sqrt{4^2 - (i \cdot 1)^2}\right)^2 \cdot 1$ or

RHS $= \pi(\sqrt{15})^2 + \pi(\sqrt{12})^2 + \pi(\sqrt{5})^2 + \pi(\sqrt{0})^2 = \sum_{i=1}^{4} \pi \left(\sqrt{4^2 - (i \cdot 1)^2}\right)^2 \cdot 1$

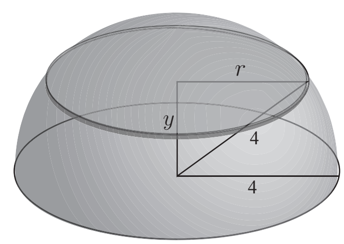
If we wanted a more accurate result, we might take thinner slices, say 8, and get:

LHS $= \sum_{i=0}^{7} \pi \left(\sqrt{4^2 - (i \cdot 0.5)^2}\right)^2 \cdot 0.5.$

and ultimately,

LHS $= \sum_{i=0}^{n-1} \pi \left(\sqrt{4^2 - (i \cdot \Delta y)^2}\right)^2 \cdot \Delta y = \sum_{i=0}^{n-1} \pi(4^2 - (i \cdot \Delta y)^2) \cdot \Delta y.$

This suggests a continuous model through integration. Recall the limit definition that $\lim_{n\to\infty} \sum_{i=0}^{n-1} f(a + i \cdot \Delta y) \cdot \Delta y = \int_a^b f(y)\,\mathrm{d}y$. The sum takes discrete values indexed by i and as these slices become thinner, $a + i\Delta y$ (summed up to b) is replaced by y ranging over the interval $[a, b]$.
In this case, $a = 0$ so $i\Delta y$ is replaced by y and we have



$$V = \int_0^4 \pi(16 - y^2)\,\mathrm{d}y$$