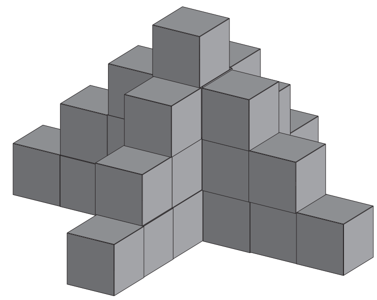
Series Problems Name:____________________

Show all relevant work!

1. Consider the pyramid of bricks shown to the right.

(a) Write a formula for the number of bricks on the n^{th} level.



(b) Find the total number of bricks needed to build this pyramid 100 rows high (don't just find the number for the 100^{th} level).

2. Determine if the sequence converges and determine the limit of each convergent sequence.

(a) $f(n) = \dfrac{n+1}{n} - \dfrac{n}{n+1}$

(b) $f(n) = \dfrac{1}{\sqrt[n]{2}}$

(c) $f(n) = \left(1 + \dfrac{2}{n}\right)^n$

(d) $f(n) = (-1)^n \left(\dfrac{9}{10}\right)^n$

3. A superball rebounds to 70% of its previous height every time it bounces. If it is dropped initially from a height of 6 feet, what is the total vertical distance it travels as it bounces up and down?

4. Write 5.4321321321. . . as a fraction.

5. Show $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$.

6. Provide an example that shows if $\sum a_n$ diverges and $\sum b_n$ diverges, $\sum a_n - b_n$ *does not* diverge.

7. Determine whether the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\ln(\ln n)}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\ln n}]$ (e) $\displaystyle\sum_{n=1}^{\infty} n!\, e^{-n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+1}$ (g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n-1)}{n+1}$ (h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{2}}$ (i) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{9}{10}\right)^n$

8. Find the Taylor Series, centered on $x = 0$, for $f(x) = \dfrac{1}{1-x^2}$.
Then determine which values of x it converges for.

9. Find the Taylor Series, centered on $x = 0$, for $f(x) = \dfrac{1}{(1-x^2)^3}$.
Then determine which values of x it converges for.

10. The Taylor series for $\arctan x$ (centered at $x = 0$) is given below.

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

We know that $\arctan(1) = \dfrac{\pi}{4}$ so using the series above we can approximate π ($\pi = 4\arctan(1)$).
Use the error bound for the series above to determine the degree of the Taylor Polynomial necessary to approximate π accurate to 6 decimal places.

11. Write a power series that has $(1, 5)$ as its interval of convergence.