

Show all relevant work!

1. Consider the wall of bricks shown to the right.

(a) Write a formula for the number of bricks in the n^{th} row.

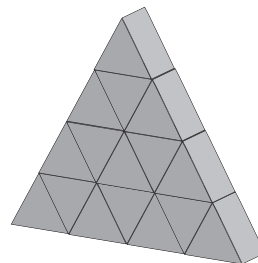
Solution:

$$1, 3, 5, \dots, 2n - 1$$

(b) Find the total number of bricks needed to build this wall 75 rows high (don't just find the number for the 75th row).

Solution:

n	1	2	3	4	...	n
S_n	1	4	9	16	...	n^2



2. Find the term rule for the sequence: 5, 8, 18, 35, 59, ...

Solution: This is a quadratic sequence so we can use methods from pre-calculus to find the formula or regression on TI-83.

n	1	2	3	4	5	...	n
t_n	5	8	18	35	59	...	$\frac{7}{2}n^2 - \frac{15}{2}n + 9$

3. Find the formula for the n^{th} partial sum, S_n , of the series $5 + 8 + 18 + 35 + 59 + \dots$

Solution:

n	1	2	3	4	5	...	n
S_n	5	13	31	66	125	...	?

$$6a = 7 \rightarrow a = \frac{7}{6}$$

$$12a + 2b = 10 \rightarrow 12\left(\frac{7}{6}\right) + 2b = 10 \rightarrow b = -2$$

$$7a + 3b + c = 8 \rightarrow 7\left(\frac{7}{6}\right) + 3(-2) + c = 8 \rightarrow c = \frac{35}{6}$$

$$a + b + c + d = 5 \rightarrow d = 0$$

$$S_n = \frac{7}{6}n^3 - 2n^2 + \frac{35}{6}n.$$

4. The graph of a geometric sequence is shown below.

(a) Find the sequence rule for the n^{th} term, t_n .

Solution:

n	1	2	3	4	5	...	n
t_n	16	12	9	?	?	...	$16 \cdot \left(\frac{3}{4}\right)^{n-1}$

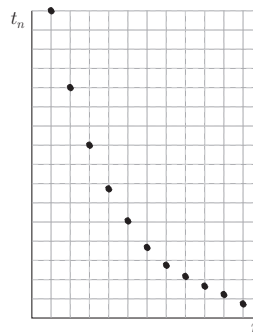
(b) Find the infinite sum of the terms in the sequence from part (a).

Solution:

$$t_1 = 16 \text{ and } r = \frac{3}{4} \text{ so } S = 16 \cdot \frac{1}{1 - \frac{3}{4}} = 16 \cdot 4 = 64.$$

5. If a geometric sequence has $t_1 = 17$ and common ratio, $r = 1.4$, find n if $t_n \approx 1889$.

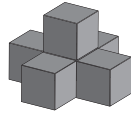
Solution: $17 \cdot (1.4)^{n-1} = 1889 \rightarrow n = 15.$



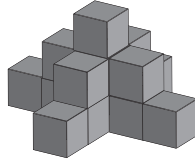
6. Find the total number of bricks needed to build the 50th figure in this sequence.



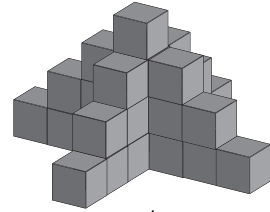
1



2



3



4

Solution: Quadratic again, note the rate of change in the rate of change is constant. We can factor the outputs for a clue, use pre-calc methods or TI-83 regression.

n	1	2	3	4	...	n
S_n	1	6	15	28	...	$n(2n-1)$

→ 50(99) = 4950

7. Find these sums:

(a) S_{85} for $1 + 13 + 25 + 37 + \dots$

(b) S_{42} for $3 + 4.5 + 6.75 + 10.125 + \dots$

(c) $\sum_{k=1}^{200} 2k + 3$

Solution:

$t_1 = 1$ and $t_{85} = 1 + (84)(12) = 1009$
 so $S_{85} = \frac{85}{2}(1 + 1009) = 42,925$

$t_1 = 3$ and $r = \frac{3}{2}$ so $t_n = 3(\frac{3}{2})^{n-1}$
 then $S_{42} = 3 \frac{1 - (\frac{3}{2})^{42}}{1 - \frac{3}{2}}$

$t_1 = 5$ and $t_{200} = 403$ so
 $S_{200} = \frac{200}{2}(5 + 403) = 20,400$

8. Determine whether the series below Converge or Diverge. If they Converge determine the value they converge to. If they Diverge say so and explain why.

(a) $12 + 9.6 + 7.68 + 6.144 + \dots$

(b) $2 + 2.2 + 2.42 + 2.662 + \dots$

Solutions:

$S = 12 \frac{1}{1-0.8} = 60$

$r = 1.1 > 1$ therefore diverges.

(c) $\sum_{k=1}^{\infty} 5 \cdot \left(\frac{2}{3}\right)^{k-1}$

(d) $4 - 3 + 2.25 - 1.6875 + \dots$

(e) $\sum_{k=1}^{\infty} (20 \cdot (0.7)^k - 5)$

Solutions:

$S = 5 \cdot \frac{1}{1-\frac{2}{3}} = 15$

$S = 4 \cdot \frac{1}{1-(-\frac{3}{4})} = \frac{16}{7}$

Diverges, $\lim_{n \rightarrow \infty} a_n = -5$.

9. Write 1.851515151... as a fraction.

Solution:

$1.851515151\dots = 1.8 + 0.051 + 0.00051 + 0.0000051 + \dots = \frac{18}{10} + 0.051 \cdot \frac{1}{1-.01}$
 $= \frac{18}{10} + \frac{51}{990} = \frac{18}{10} + \frac{17}{330} = \frac{611}{330}$

10. Evaluate $\sum_{k=1}^{\infty} k^2$

Solution: Again, this assumes we saw this pattern in pre-calc/calc 1. The rate of change in the rate of change in the rate of change is constant so the original sequence must be cubic.

n	1	2	3	4	...	n
S_n	1	5	14	30	...	?

$6a = 2 \rightarrow a = \frac{1}{3}$
 $12a + 2b = 5 \rightarrow b = \frac{1}{2}$
 $7a + 3b + c = 4 \rightarrow c = \frac{1}{6}$
 $a + b + c + d = 1 \rightarrow d = 0$

→ $S_n = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$

11. Find the infinite sum, $1 + 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^4 + \dots$

Solution:

$$\begin{aligned}
 1 + 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^4 + \dots &= \\
 1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \dots &= 1 \cdot \frac{1}{1 - \frac{1}{3}} = \frac{3}{2} \\
 \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \dots &= \frac{1}{3} \cdot \frac{1}{1 - \frac{1}{3}} = \frac{1}{2} \\
 \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \dots &= \left(\frac{1}{3}\right)^2 \cdot \frac{1}{1 - \frac{1}{3}} = \frac{1}{6} \\
 \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \dots &= \left(\frac{1}{3}\right)^3 \cdot \frac{1}{1 - \frac{1}{3}} = \frac{1}{18} \\
 \vdots & \\
 S = \frac{3}{2} + \frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \dots &= \frac{3}{2} \cdot \frac{1}{1 - \frac{1}{3}} \\
 &= \frac{9}{4}
 \end{aligned}$$