Math 252

Log Project

Complete the following project:

When we consider the antiderivative of $f(t) = t^r$ there is a general result for all cases except where $r = -1$. This is often confusing and for this reason among others, many books that begin with integral calculus *define* the integral

$$\int_1^x \frac{1}{t}\,\mathrm{d}t = \ln x$$

To see that this is a reasonable assertion, we will demonstrate that the properties of the integral on the left match those of the function on the right.

Problems to solve:

(a) Building Intuition: let $F(r) = \int_1^2 t^r\,\mathrm{d}t$, with $r \in \mathbb{R}$, $r \neq 1$. Evaluate $F(r)$ for values of r close to -1. (You will want to use a calculator for this. Include a table of several values with your results.) Do the values of $F(r)$ seem to approach a limit as $r \to -1$? Do you recognize the limit? Replace the upper bound of the integral with 2, 3, 4, and 10 and repeat the same analysis in each case.

(b) Let b be a fixed positive number. For $r \in \mathbb{R}, r \neq -1$, redefine the function F as $F(r) = \int_1^b t^r\,\mathrm{d}t$. Evaluate the integral to find a simpler formula for $F(r)$.

(c) Show that $F(r) = \int_1^b t^r\,\mathrm{d}t$ is a continuous function. Give a good justification for your conclusion. (Read about continuity and polynomial functions.)

(d) How should we define $F(-1)$ so that F is continuous at $r = -1$? Show that this value makes F continuous at $r = -1$. Reread the introductory paragraph and comment on the objective of this project. Explain your results to part (a) in light of your discoveries in the part.

(e) Show that $L(x) = \int_1^x \frac{1}{t}\,\mathrm{d}t$ satisfies the fundamental properties of logarithmic functions, (i) L(1) = 0, and (ii) $L(ab) = L(a) + L(b)$. (To do this you will need properties of integrals and a clever substitution).

Completed Project:
When you have done the work necessary to complete the project, you need to prepare it in written form. The paper you turn in should have a mix of equations formulas and prose to support your conclusions. Use complete sentences, good grammar and correct punctuation. The prose should be written in order to convey to the reader an explanation of what you have done. It should be written in such a way that it can be read and understood by anyone who knows the material in this course. You will be graded on your written presentation as well as the mathematical content.

Adapted from Cohen, Gaughan, et al. *Student Research Projects in Calculus*; MAA: Washington, D.C., 1991