Exam 1 Name:__________________________________

YOU MAY USE A CALCULATOR TO COMPUTE SOLUTIONS BUT SHOW YOUR SET-UPS.

Show all relevant work!

1 A tank has a height of 10 feet. The area of the horizontal cross section of the tank at height h feet is given by the function A , where $A(h)$ is measured in square feet. The function A is continuous and decreases as h increases. Selected values for $A(h)$ are given in the table below.

h (feet)	0	2	5	10
$A(h)$ (square feet)	50.3	14.4	6.5	2.9

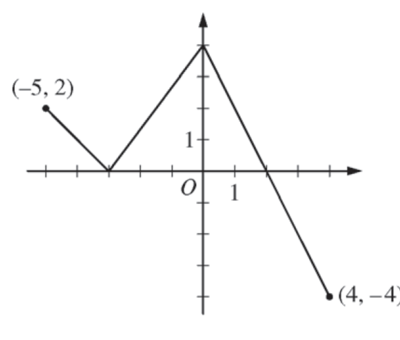
(a) Use a trapezoidal Riemann sum with the three subintervals indicated by the data in the table to approximate the volume of the tank. Indicate units of measure. **(Ans: 119.6 ft^3)**

(b) Does the approximation in part (a) overestimate or underestimate the volume of the tank? Explain your reasoning.

2 Use methods of calculus to find $\displaystyle\int \frac{1}{P^2 - 2P}\,\mathrm{d}P$. **(Ans:** $\frac{1}{2}\ln\left|\frac{P-2}{P}\right|$**)**

3 A Ferrari 360 Modena going at 60 mph (88 ft/sec) can stop in 110 feet. Find the acceleration (deceleration) assuming it is constant. **(Ans: –35.2)**

4 The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \displaystyle\int_{-3}^{x} f(t)\,\mathrm{d}t$



Graph of f

(a) Find $g(3)$ **(Ans: 9)**

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer. **(Ans: (–5, –3) and (0, 2))**

5 $F(x) = \displaystyle\int_0^{x^2} e^{-t^3}\,\mathrm{d}t$ is defined on $(-\infty, \infty)$. Show F has a local minimum at (0, 0).

6. Perform the first step in substitution for each of the indefinite integrals below. In each case identify the correct substitution and carry out the change of variables, rewriting (but not evaluating) the new integral.

(a) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x$ (b) $\displaystyle\int \frac{1}{\sqrt{1+x^2}}\,\mathrm{d}x$

7. Throughout much of the 20^{th} century, the yearly consumption of electricity in the US increased exponentially at a continuous rate of 7% per year. Assuming the trend continues, the model for the yearly consumption of electricity (in millions of megawatt-hours) as a function of time since 1900 is given by $E(t) = 1.4e^{0.07t}$.

Use the Fundamental Theorem to help you find the average yearly electrical consumption throughout the 20^{th} century (1900 – 2000). Be sure to include units in your answer. **(Ans: 219.13)**

8. Consider the indefinite integral $\displaystyle\int x^n e^x\,\mathrm{d}x$.

(a) Show $\displaystyle\int x^n e^x\,\mathrm{d}x = x^n e^x - n\int x^{n-1}e^x\,\mathrm{d}x$

(b) Use the formula in part (a) to find $\displaystyle\int x^3 e^x\,\mathrm{d}x$.