

## Summary of Convergence Tests for Series

Test	Series	Convergence or Divergence	Comments
$n^{\text{th}}$ term test (or the zero test)	$\sum a_n$	Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$	Inconclusive if $\lim_{n \rightarrow \infty} a_n = 0$ .
Geometric series	$\sum_{n=0}^{\infty} ax^n$ (or $\sum_{n=1}^{\infty} ax^{n-1}$ )	Converges to $\frac{a}{1-x}$ only if $ x  < 1$ Diverges if $ x  \geq 1$	Useful for comparison tests if the $n^{\text{th}}$ term $a_n$ of a series is similar to $ax^n$ .
$p$ -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ Diverges if $p \leq 1$	Useful for comparison tests if the $n^{\text{th}}$ term $a_n$ of a series is similar to $\frac{1}{n^p}$ .
Integral	$\sum_{n=c}^{\infty} a_n$ ( $c \geq 0$ ) $a_n = f(n)$ for all $n$	Converges if $\int_c^{\infty} f(x) dx$ converges Diverges if $\int_c^{\infty} f(x) dx$ diverges	The function $f$ obtained from $a_n = f(n)$ must be continuous, positive, decreasing and readily integrable for $x \geq c$ .
Comparison	$\sum a_n$ and $\sum b_n$ with $0 \leq a_n \leq b_n$ for all $n$	$\sum b_n$ converges $\implies \sum a_n$ converges $\sum a_n$ diverges $\implies \sum b_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a $p$ -series.
Limit Comparison	$\sum a_n$ and $\sum b_n$ with $a_n, b_n > 0$ for all $n$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$	$\sum b_n$ converges $\implies \sum a_n$ converges $\sum b_n$ diverges $\implies \sum a_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a $p$ -series. To find $b_n$ consider only the terms of $a_n$ that have the greatest effect on the magnitude.
Ratio	$\sum a_n$ with $\lim_{n \rightarrow \infty} \frac{ a_{n+1} }{ a_n } = L$	Converges (absolutely) if $L < 1$ Diverges if $L > 1$ or if $L$ is infinite	Inconclusive if $L = 1$ . Useful if $a_n$ involves factorials or $n^{\text{th}}$ powers.
Root	$\sum a_n$ with $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = L$	Converges (absolutely) if $L < 1$ Diverges if $L > 1$ or if $L$ is infinite	Test is inconclusive if $L = 1$ . Useful if $a_n$ involves $n^{\text{th}}$ powers.
Absolute Value $\sum  a_n $	$\sum a_n$	$\sum  a_n $ converges $\implies \sum a_n$ converges	Useful for series containing both positive and negative terms.
Alternating series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ ( $a_n > 0$ )	Converges if $0 < a_{n+1} < a_n$ for all $n$ and $\lim_{n \rightarrow \infty} a_n = 0$	Applicable only to series with alternating terms.

# Series Convergence Flow Chart

