

YOU MAY USE A CALCULATOR TO COMPUTE SOLUTIONS BUT SHOW YOUR SET-UPS.

Show all relevant work!

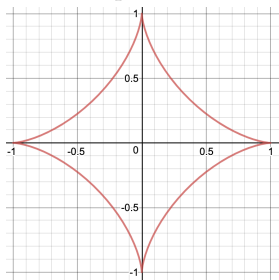
- ① Set up (but don't evaluate) integrals for the volume of the surface of revolution formed by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about . . .

(a) the x -axis

(b) the line $x = 1$

- ② Set up (but don't evaluate) an integral for the volume of the surface of revolution formed by rotating the region bounded by $y = 2^{-x}$ about the y -axis.

- ③ Find the perimeter of the parametric curve given by $x = \cos^3 t$, $y = \sin^3 t$ for $0 \leq t \leq 2\pi$



- ④ Rotating the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$ (see Fig. 1) about the x -axis generates a star-shaped solid (see Fig 2). Compute its volume.

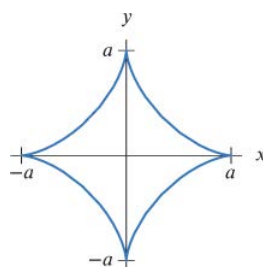


Figure 1: Hypocycloid

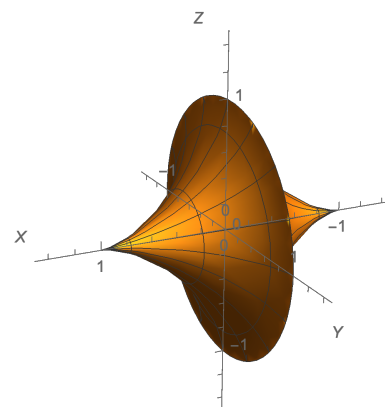


Figure 2: Revolved about x -axis

- ⑤ A tree trunk has a circular cross-section at every height; its circumference is given in the following table. Estimate the volume of the tree trunk using the trapezoid rule.

Height (in)	0	20	40	60	80	100	120
Circumference (in)	26	22	19	14	6	3	1

Table 1: Tree measurements

- ⑥ A transmission line is strung between two power poles, 200 feet apart (see fig). The shape of the cable is modeled by $y = 50 \cosh\left(\frac{x}{200}\right)$. Find the length of the cable strung between the two poles.

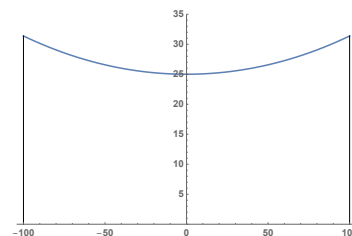


Figure 3: Transmission line

- ⑦ Find the volume of the solid whose base is the region bounded by the parabolas $y = x^2$ and $y = 8 - x^2$ and whose cross-sections perpendicular to the y -axis are semicircles.

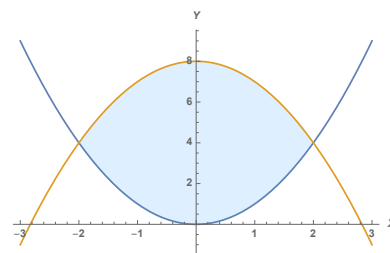


Figure 4: Region

- ⑧ The design of boats is based on Archimedes' Principle, which states that the buoyant force on an object in water is equal to the weight of the water displaced. Suppose you want to build a sailboat whose hull is parabolic with cross-section $y = ax^2$, where a is a constant. Your boat will have length L and its maximum draft (the maximum vertical depth of any point of the boat beneath the water line) will be H . See Figure 5. Every cubic meter of water weighs 10,000 newtons. What is the maximum possible weight for your boat and cargo?

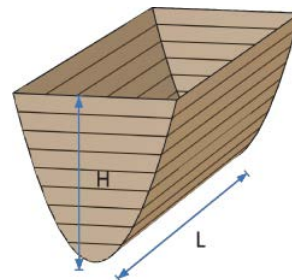


Figure 5: Hull