Chp. 8 Closing Problems

Show all relevant work!

Some useful(?) formulas:

$C = 2\pi r$ $A = \pi r^2$ $SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$ $\cosh^2 x - \sinh^2 x = 1$

Arclength: $\ell = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$ and $\ell = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$ Area $= \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\, d\theta$ $\overline{x} = \frac{\int x\, dm}{\int dm}$

Don't Panic

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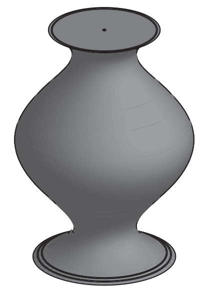
1. A block of maple is turned on a lathe to produce the solid sculpture shown on the right. Maple has a weight density of 0.2 pounds per cubic inch. The table below shows diameter measurements of the piece at height increments of 2 inches.



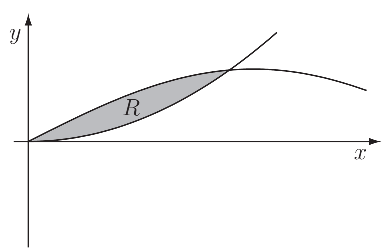
Height (in):	0	2	4	6	8	10
Diameter (in):	3.9	3.0	4.5	4.9	3.3	3.6

(a) Use horizontal slices and a Right Hand sum with increments of 2 inches to approximate the weight of the sculpture.

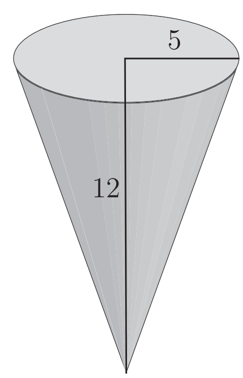
(b) Use the Right Hand sum with increments of 2 inches to help you determine the vertical center of mass of the sculpture (we will assume the horizontal center of mass (weight) lies along the axis of rotation).

2. A solid is formed by revolving the region bounded by $y = 4 - 0.2x^2$ and the $x-$ and $y-$axes about the $y-$axis. Find the center of mass of the solid assuming its density varies with the square of its distance above the plane of the $x-$axis and the the center lies along the axis of rotation.

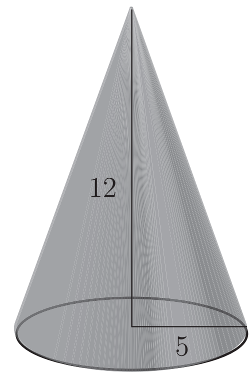
3. Consider the region, R, bounded by $y = 2\sin x$, and $y = x^2$. Suppose we rotate the region R about the x-axis. Assuming uniform mass density, δ kg/m^3, and all lengths in meters, determine the center of mass (along the axis of rotation) for this solid. Show any relevant integrals.



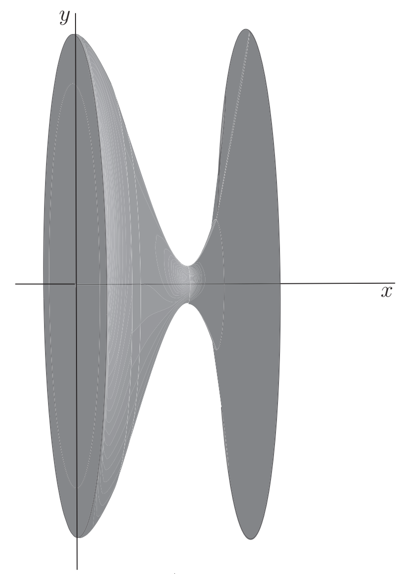
4. A conical tank of radius 5 feet and height 12 feet is filled with water. Find the work done by gravity in emptying the tank through the bottom. Assume the weight density of the water is 62.4 lbs/ft^3.



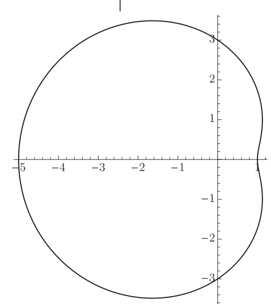
5. Repeat number (4) if the tank is turned upside down (but still drained by gravity down through the bottom).



6. Consider the region bounded by $y = x^3 - 3x^2 + 5$, the x-axis, the y-axis, and the line $x = 3$. Suppose we rotate this region about the x-axis. Assuming uniform mass density, δ kg/m^3, and all lengths in meters, determine the center of mass (along the axis of rotation) for this solid.

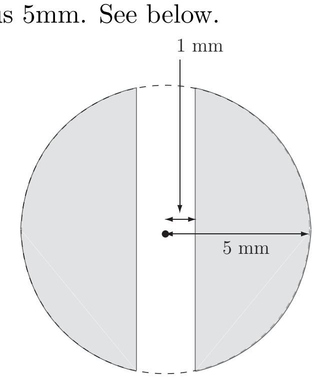


7. Find the arc length of the cardioid $r = 3 - 2\cos\theta$.



8. A bead is made by drilling a cylindrical hole of radius 1mm through a sphere of radius 5mm. See below.

Set up (and evaluate) an integral representing the volume of the bead.



9. The population of a city varies inversely with the distance from the city center according to the density function, $\delta r = \frac{20}{r+1}$ where δ is measured in thousands of people per square kilometer. Assuming the city is contained within a 3km radius, find the population of the city.