## REVIEW EXERCISES AND PROBLEMS FOR CHAPTER 7 EXERCISES

- For Exercises 1-4, find an antiderivative.

1. 

$q(t)=(t+1)^{2}$
ANSWER $\oplus$
WORKED SOLUTION $\oplus$
2.
$p(\theta)=2 \sin (2 \theta)$
3.
$f(x)=5^{x}$
ANSWER $\oplus$
4.
$r(t)=e^{t}+5 e^{5 t}$
-For Exercises 5-110, evaluate the following integrals. Assume $a, b, c$, and $k$ are constants. Exercises 7-69 can be done without an integral table, as can some of the later problems.
5.

$$
\int(3 w+7) d w
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
6.

$$
\int e^{2 r} d r
$$

7. 

$$
\int \sin t d t
$$

## ANSWER ${ }^{\oplus}$

8. 

$$
\int \cos 2 t d t
$$

9. 

$$
\int e^{5 z} d z
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
10.

$$
\int \cos (x+1) d x
$$

11. 

$$
\int \sin 2 \theta d \theta
$$

## ANSWER ${ }^{\oplus}$

12. 

$$
\int\left(x^{3}-1\right)^{4} x^{2} d x
$$

13. 

$$
\int\left(x^{3 / 2}+x^{2 / 3}\right) d x
$$

```
ANSWER © 
WORKED SOLUTION © 
```

14. 

$$
\int\left(e^{x}+3^{x}\right) d x
$$

15. 

$$
\int \frac{1}{e^{z}} d z
$$

ANSWER ${ }^{\oplus}$
16.

$$
\int\left(\frac{4}{x^{2}}-\frac{3}{x^{3}}\right) d x
$$

17. 

$$
\int \frac{x^{3}+x+1}{x^{2}} d x
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
18.

$$
\int \frac{(1+\ln x)^{2}}{x} d x
$$

19. 

$$
\int t e^{t^{2}} d t
$$

## ANSWER $\oplus$

20. 

$$
\int x \cos x d x
$$

21. 

$$
\int x^{2} e^{2 x} d x
$$

```
ANSWER \(\oplus\)
WORKED SOLUTION \(\oplus\)
```

22. 

$$
\int x \sqrt{1-x} d x
$$

23. 

$$
\int y \ln y d y
$$

24. 

$$
\int y \sin y d y
$$

25. 

$$
\int(\ln x)^{2} d x
$$

## ANSWER © $\uparrow$

WORKED SOLUTION $\oplus$
26.

$$
\int e^{0.5-0.3 t} d t
$$

27. 

$$
\int \sin ^{2} \theta \cos \theta d \theta
$$

## ANSWER $\oplus$

28. 

$$
\int x \sqrt{4-x^{2}} d x
$$

29. 

$$
\int \frac{(u+1)^{3}}{u^{2}} d u
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
30.

$$
\int \frac{\cos \sqrt{y}}{\sqrt{y}} d y
$$

31. 

$$
\int \frac{1}{\cos ^{2} z} d z
$$

## ANSWER ${ }^{\oplus}$

32. 

$$
\int \cos ^{2} \theta d \theta
$$

33. 

$$
\int t^{10}(t-10) d t
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
34.

$$
\int \tan (2 x-6) d x
$$

35. 

$$
\int \frac{(\ln x)^{2}}{x} d x
$$

## ANSWER $\oplus$

36. 

$$
\int \frac{(t+2)^{2}}{t^{3}} d t
$$

37. 

$$
\int\left(x^{2}+2 x+\frac{1}{x}\right) d x
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
38.

$$
\int \frac{t+1}{t^{2}} d t
$$

39. 

$$
\int t e^{t^{2}+1} d t
$$

## ANSWER $\oplus$

40. 

$$
\int \tan \theta d \theta
$$

41. 

$$
\int \sin (5 \theta) \cos (5 \theta) d \theta
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
42.

$$
\int \frac{x}{x^{2}+1} d x
$$

43. 

$$
\int \frac{d z}{1+z^{2}}
$$

## ANSWER $\oplus$

44. 

$$
\int \frac{d z}{1+4 z^{2}}
$$

45. 

$$
\int \cos ^{3} 2 \theta \sin 2 \theta d \theta
$$

```
ANSWER © 
WORKED SOLUTION ©
```

46. 

$$
\int \sin 5 \theta \cos ^{3} 5 \theta d \theta
$$

47. 

$$
\int \sin ^{3} z \cos ^{3} z d z
$$

## ANSWER ${ }^{\oplus}$

48. 

$$
\int t(t-10)^{10} d t
$$

49. 

$$
\int \cos \theta \sqrt{1+\sin \theta} d \theta
$$

## ANSWER © $\dagger$ <br> WORKED SOLUTION ${ }^{+}$

50. 

$$
\int x e^{x} d x
$$

51. 

$$
\int t^{3} e^{t} d t
$$

## ANSWER ${ }^{\oplus}$

52. 

$$
\int_{1}^{3} x\left(x^{2}+1\right)^{70} d x
$$

53. 

$$
\int(3 z+5)^{3} d z
$$

```
ANSWER © 
WORKED SOLUTION ©
```

54. 

$$
\int \frac{d u}{9+u^{2}}
$$

55. 

$$
\int \frac{\cos w}{1+\sin ^{2} w} d w
$$

## ANSWER ${ }^{\oplus}$

56. 

$$
\int \frac{1}{x} \tan (\ln x) d x
$$

57. 

$$
\int \frac{1}{x} \sin (\ln x) d x
$$

```
ANSWER © 
WORKED SOLUTION © 
```

58. 

$$
\int \frac{w d w}{\sqrt{16-w^{2}}}
$$

59. 

$$
\int \frac{e^{2 y}+1}{e^{2 y}} d y
$$

## ANSWER $\oplus$

60. 

$$
\int \frac{\sin w d w}{\sqrt{1-\cos w}}
$$

61. 

$$
\int \frac{d x}{x \ln x}
$$

62. 

$$
\int \frac{d u}{3 u+8}
$$

63. 

$$
\int \frac{x \cos \sqrt{x^{2}+1}}{\sqrt{x^{2}+1}} d x
$$

## ANSWER ${ }^{\oplus}$

64. 

$$
\int \frac{t^{3}}{\sqrt{1+t^{2}}} d t
$$

65. 

$$
\int u e^{k u} d u
$$

## ANSWER ©

## WORKED SOLUTION $\oplus$

66. 

$$
\int(w+5)^{4} w d w
$$

67. 

$$
\int e^{\sqrt{2} x+3} d x
$$

## ANSWER $\oplus$

68. 

$$
\int\left(e^{x}+x\right)^{2} d x
$$

69. 

$$
\int u^{2} \ln u d u
$$

```
ANSWER ¢
WORKED SOLUTION +
```

70. 

$$
\int \frac{5 x+6}{x^{2}+4} d x
$$

71. 

$$
\int \frac{1}{\sin ^{3}(2 x)} d x
$$

## ANSWER $\dagger$

72. 

$$
\int \frac{d r}{r^{2}-100}
$$

73. 

$$
\int y^{2} \sin (c y) d y
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
74.

$$
\int e^{-c t} \sin k t d t
$$

75. 

$$
\int e^{5 x} \cos (3 x) d x
$$

## ANSWER ${ }^{\oplus}$

76. 

$$
\int\left(x^{\sqrt{k}}+\sqrt{k}^{x}\right) d x
$$

77. 

$$
\int \sqrt{3+12 x^{2}} d x
$$

```
ANSWER © 
WORKED SOLUTION `
```

78. 

$$
\int \frac{1}{\sqrt{x^{2}-3 x+2}} d x
$$

79. 

$$
\int \frac{x^{3}}{x^{2}+3 x+2} d x
$$

## ANSWER $\oplus$

80. 

$$
\int \frac{x^{2}+1}{x^{2}-3 x+2} d x
$$

81. 

$$
\int \frac{d x}{a x^{2}+b x}
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
82.

$$
\int \frac{a x+b}{a x^{2}+2 b x+c} d x
$$

83. 

$$
\int\left(\frac{x}{3}+\frac{3}{x}\right)^{2} d x
$$

## ANSWER ${ }^{\oplus}$

84. 

$$
\int \frac{2^{t}}{2^{t}+1} d t
$$

85. 

$$
\int 10^{1-x} d x
$$

ANSWER ©
WORKED SOLUTION $\oplus$
86.

$$
\int\left(x^{2}+5\right)^{3} d x
$$

87. 

$$
\int v \arcsin v d v
$$

## ANSWER $\oplus$

88. 

$$
\int \sqrt{4-x^{2}} d x
$$

89. 

$$
\int \frac{z^{3}}{z-5} d z
$$

```
ANSWER ©
WORKED SOLUTION © 
```

90. 

$$
\int \frac{\sin w \cos w}{1+\cos ^{2} w} d w
$$

91. 

$$
\int \frac{1}{\tan (3 \theta)} d \theta
$$

92. 

$$
\int \frac{x}{\cos ^{2} x} d x
$$

93. 

$$
\int \frac{x+1}{\sqrt{x}} d x
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
94.

$$
\int \frac{x}{\sqrt{x+1}} d x
$$

95. 

$$
\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}} d x
$$

## ANSWER ${ }^{\oplus}$

96. 

$$
\int \frac{e^{2 y}}{e^{2 y}+1} d y
$$

97. 

$$
\int \frac{z}{\left(z^{2}-5\right)^{3}} d z
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
98.

$$
\int \frac{z}{(z-5)^{3}} d z
$$

99. 

$$
\int \frac{(1+\tan x)^{3}}{\cos ^{2} x} d x
$$

## ANSWER $\oplus$

100. 

$$
\int \frac{(2 x-1) e^{x^{2}}}{e^{x}} d x
$$

101. 

$$
\int(2 x+1) e^{x^{2}} e^{x} d x
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
102.

$$
\int \sqrt{y^{2}-2 y+1}(y-1) d y
$$

103. 

$$
\int \sin x(\sqrt{2+3 \cos x}) d x
$$

## ANSWER $\oplus$

104. 

$$
\int\left(x^{2}-3 x+2\right) e^{-4 x} d x
$$

105. 

$$
\int \sin ^{2}(2 \theta) \cos ^{3}(2 \theta) d \theta
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
106.

$$
\int \cos (2 \sin x) \cos x d x
$$

107. 

$$
\int(x+\sin x)^{3}(1+\cos x) d x
$$

## ANSWER ${ }^{\oplus}$

108. 

$$
\int\left(2 x^{3}+3 x+4\right) \cos (2 x) d x
$$

109. 

$$
\int \sinh ^{2} x \cosh x d x
$$

## ANSWER $\oplus$

## WORKED SOLUTION $\oplus$

110. 

$$
\int(x+1) \sinh \left(x^{2}+2 x\right) d x
$$

- For Exercises 111-124, evaluate the definite integrals using the Fundamental Theorem of Calculus and check your answers numerically.

111. 

$$
\int_{0}^{1} x\left(1+x^{2}\right)^{20} d x
$$

## ANSWER $\oplus$

112. 

$$
\int_{4}^{1} x \sqrt{x^{2}+4} d x
$$

113. 

$$
\int_{0}^{\pi} \sin \theta(\cos \theta+5)^{7} d \theta
$$

```
ANSWER © 
WORKED SOLUTION © 
```

114. 

$$
\int_{0}^{1} \frac{x}{1+5 x^{2}} d x
$$

115. 

$$
\int_{1}^{2} \frac{x^{2}+1}{x} d x
$$

## ANSWER $\oplus$

116. 

$$
\int_{1}^{3} \ln \left(x^{3}\right) d x
$$

117. 

$$
\int_{1}^{e}(\ln x)^{2} d x
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
118.

$$
\int_{-\pi}^{\pi} e^{2 x} \sin 2 x d x
$$

119. 

$$
\int_{0}^{10} z e^{-z} d z
$$

ANSWER $\oplus$
120.

$$
\int_{-\pi / 3}^{\pi / 4} \sin ^{3} \theta \cos \theta d \theta
$$

121. 

$$
\int_{1}^{8} \frac{e \sqrt[3]{x}}{\sqrt[3]{x^{2}}} d x
$$

## WORKED SOLUTION $\oplus$

122. 

$$
\int_{0}^{1} \frac{d x}{x^{2}+1}
$$

123. 

$$
\int_{-\pi / 4}^{\pi / 4} \cos ^{2} \theta \sin ^{5} \theta d \theta
$$

## ANSWER © ${ }^{+}$

124. 

$$
\int_{-2}^{0} \frac{2 x+4}{x^{2}+4 x+5} d x
$$

125. 

Use partial fractions on $\frac{1}{x^{2}-1}$ to find $\int \frac{1}{x^{2}-1} d x$.
ANSWER $\oplus$
WORKED SOLUTION $\oplus$
126.
(a)

Use partial fractions to find $\int \frac{1}{x^{2}-x} d x$
(b)

Show that your answer to part a agrees with the answer you get by using the integral tables.
127.

Use partial fractions to find $\int \frac{1}{x(L-x)} d x$, where $L$ is constant.
ANSWER $\oplus$

- Evaluate the integrals in Exercises 128-139 using partial fractions or a trigonometric substitution ( $a$ and $b$ are positive constants).

128. 

$$
\int \frac{1}{(x-2)(x+2)} d x
$$

129. 

$$
\begin{aligned}
& \int \frac{1}{\sqrt{25-x^{2}}} d x \\
& \text { ANSWER } \oplus \\
& \text { WORKED SOLUTION } \oplus+
\end{aligned}
$$

130. 

$$
\int \frac{1}{x(x+5)} d x
$$

131. 

$$
\int \frac{1}{\sqrt{1-9 x^{2}}} d x
$$

## ANSWER $\oplus$

132. 

$$
\int \frac{2 x+3}{x(x+2)(x-1)} d x
$$

133. 

$$
\int \frac{3 x+1}{x\left(x^{2}-1\right)} d x
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
134.

$$
\int \frac{1+x^{2}}{x(1+x)^{2}} d x
$$

135. 

$$
\int \frac{1}{x^{2}+2 x+2} d x
$$

## ANSWER ${ }^{\oplus}$

136. 

$$
\int \frac{1}{x^{2}+4 x+5} d x
$$

137. 

$$
\int \frac{1}{\sqrt{a^{2}-(b x)^{2}}} d x
$$

## ANSWER © ${ }^{+}$

WORKED SOLUTION $\oplus$
138.

$$
\int \frac{\cos x}{\sin ^{3} x+\sin x} d x
$$

139. 

$$
\int \frac{e^{x}}{e^{2 x}-1} d x
$$

ANSWER ${ }^{\oplus}$
■ Calculate the integrals in Exercises 140-143, if they converge. You may calculate the limits by appealing to the dominance of one function over another, or by l'Hopital's rule. 140.

$$
\int_{0}^{4} \frac{d x}{\sqrt{16-x^{2}}}
$$

141. 

$$
\int_{0}^{3} \frac{5}{x^{2}} d x
$$

ANSWER $\oplus$
WORKED SOLUTION ${ }^{+}$
142.

$$
\int_{0}^{2} \frac{1}{x-2} d x
$$

143. 

$$
\int_{0}^{8} \frac{1}{\sqrt[3]{8-x}} d x
$$

- For Exercises 144-157 decide if the integral converges or diverges. If the integral converges, find its value or give a bound on its value.

144. 

$$
\int_{4}^{\infty} \frac{d t}{t^{3 / 2}}
$$

145. 

$$
\int_{10}^{\infty} \frac{d x}{x \ln x}
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
146.

$$
\int_{0}^{\infty} w e^{-w} d w
$$

147. 

$$
\int_{-1}^{1} \frac{1}{x^{4}} d x
$$

## ANSWER $\oplus$

148. 

$$
\int_{-\pi / 4}^{\pi / 4} \tan \theta d \theta
$$

149. 

$$
\int_{2}^{\infty} \frac{1}{4+z^{2}} d z
$$

```
ANSWER © 
WORKED SOLUTION © 
```

150. 

$$
\int_{10}^{\infty} \frac{1}{z^{2}-4} d z
$$

151. 

$$
\int_{-5}^{10} \frac{d t}{\sqrt{t+5}}
$$

## ANSWER $\oplus$

152. 

$$
\int_{0}^{\pi / 2} \frac{1}{\sin \phi} d \phi
$$

153. 

$$
\int_{0}^{\pi / 4} \tan 2 \theta d \theta
$$

ANSWER $\oplus$
WORKED SOLUTION $\dagger$
154.

$$
\int_{1}^{\infty} \frac{x}{x+1} d x
$$

155. 

$$
\int_{0}^{\infty} \frac{\sin ^{2} \theta}{\theta^{2}+1} d \theta
$$

ANSWER $\oplus$
156.

$$
\int_{0}^{\pi} \tan ^{2} \theta d \theta
$$

157. 

$$
\int_{0}^{1}(\sin x)^{-3 / 2} d x
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$

## PROBLEMS

- In Problems 158-160, find the exact area.

158. 

Under $y=\left(e^{x}\right)^{2}$ for $0 \leq x \leq 1$.
159.

Between $y=\left(e^{x}\right)^{3}$ and $y=\left(e^{x}\right)^{2}$ for $0 \leq x \leq 3$.
ANSWER $\oplus$
160.

Between $y=e^{x}$ and $y=5 e^{-x}$ and the $y$-axis.
161.

The curves $y=\sin x$ and $y=\cos x$ cross each other infinitely often. What is the area of the region bounded by these two curves between two consecutive crossings?
ANSWER $\oplus$
WORKED SOLUTION $\oplus$
162.

Evaluate $\int_{0}^{2} \sqrt{4-x^{2}} d x$ using its geometric interpretation.
■In Problems 163-164, find a substitution $w$ and constants $k, p$ so that the integral has the form $\int k w^{p} d w$ 163.

$$
\int 3 x^{4} \sqrt{3 x^{5}+2} d x
$$

## ANSWER © ${ }^{+}$

164. 

$$
\int \frac{5 \sin (3 \theta) d \theta}{\cos ^{3}(3 \theta)}
$$

- In Problems 165-168, give the substitution and the values of any constants to rewrite the integral in the desired form.

165. 

$\int \frac{d x}{(2 x-3)(3 x-2)}$ as $\int\left(\frac{A}{2 x-3}+\frac{B}{3 x-2}\right) d x$
ANSWER ${ }^{\oplus}$
WORKED SOLUTION $\oplus$
166.
$\int\left(x^{2}+x\right) \cos (0.5 x-1) d x$ as $\int p(u) \cos (u) d u$ where $p(u)$ is a polynomial
167.
$\int x^{3} e^{-x^{2}} d x$ as $\int k u e^{u} d u$

## ANSWER ${ }^{\oplus}$

168. 

$\int \frac{\cos ^{4}(\sqrt{x}) \sin \sqrt{x} d x}{\sqrt{x}}$ as $\int k u^{n} d u$

- In Problems 169-172, explain why the following pairs of antiderivatives are really, despite their apparent dissimilarity, different expressions of the same problem. You do not need to evaluate the integrals. 169.

$$
\int \frac{1}{\sqrt{1-x^{2}}} d x \text { and } \int \frac{x d x}{\sqrt{1-x^{4}}}
$$

## ANSWER $\oplus$

## WORKED SOLUTION $\oplus$

170. 

$$
\int \frac{d x}{x^{2}+4 x+4} \text { and } \int \frac{x}{\left(x^{2}+1\right)^{2}} d x
$$

171. 

$$
\int \frac{x}{1-x^{2}} d x \text { and } \int \frac{1}{x \ln x} d x
$$

## ANSWER ${ }^{\oplus}$

172. 

$$
\int \frac{x}{x+1} d x \text { and } \int \frac{1}{x+1} d x
$$

- In Problems 173-174, show the two integrals are equal using a substitution.

173. 

$$
\int_{0}^{2} e^{-w^{2}} d w=\int_{0}^{1} 2 e^{-4 x^{2}} d x
$$

ANSWER ${ }^{\oplus}$

## WORKED SOLUTION $\oplus$

174. 

$$
\int_{0}^{3} \frac{\sin t}{t} d t=\int_{0}^{1} \frac{\sin 3 t}{t} d t
$$

175. 

A function is defined by $f(t)=t^{2}$ for $0 \leq t \leq 1$ and $f(t)=2-t$ for $1<t \leq 2$. Compute $\int_{0}^{2} f(t) d t$. ANSWER $\oplus$
176.
(a)

Find $\int(x+5)^{2} d x$ in two ways:
(i)

By multiplying out
(ii)

By substituting $w=x+5$
(b)

Are the results the same? Explain.
177.

Suppose $\int_{-1}^{1} h(z) d z=7$, and that $h(z)$ is even. Calculate the following:
(a)

$$
\int_{0}^{1} h(z) d z
$$

## ANSWER $\oplus$

WORKED SOLUTION $\oplus$
(b)

$$
\int_{-4}^{-2} 5 h(z+3) d z
$$

ANSWER $\oplus$
WORKED SOLUTION $\oplus$
178.

Find the average (vertical) height of the shaded area in Figure 7.28.


Figure 7.28
179.

Find the average (horizontal) width of the shaded area in Figure 7.28.

## ANSWER ${ }^{\oplus}$

180. 

(a)

Find the average value of the following functions over one cycle:
(i)
$f(t)=\cos t$
(ii)
$g(t)=|\cos t|$
(iii)
$k(t)=(\cos t)^{2}$
(b)

Write the averages you have just found in ascending order. Using words and graphs, explain why the averages come out in the order they do.
181.

What, if anything, is wrong with the following calculation?

$$
\int_{-2}^{2} \frac{1}{x^{2}} d x=-\left.\frac{1}{x}\right|_{-2} ^{2}=-\frac{1}{2}-\left(-\frac{1}{-2}\right)=-1
$$

()

## ANSWER $\oplus$

## WORKED SOLUTION $\oplus$

182. 

Let

$$
E(x)=\int \frac{e^{x}}{e^{x}+e^{-x}} d x \text { and } F(x)=\int \frac{e^{-x}}{e^{x}+e^{-x}} d x
$$

()
(a)

Calculate $E(x)+F(x)$.
(b)

Calculate $E(x)-F(x)$.
(c)

Use your results from parts a and b to calculate $E(x)$ and $F(x)$.
183.

Using Figure 7.29, put the following approximations to the integral $\int_{a}^{b} f(x) d x$ and its exact value in order from smallest to largest: LEFT(5), LEFT(10), RIGHT(5), RIGHT(10), MID(10), TRAP(10), Exact value


Figure 7.29
ANSWER $\oplus$
184.

You estimate $\int_{0}^{0.5} f(x) d x$ by the trapezoid and midpoint rules with 100 steps. Which of the two estimates is an overestimate, and which is an underestimate, of the true value of the integral if
(a)
$f(x)=1+e^{-x}$
(b)

$$
f(x)=e^{-x^{2}}
$$

(c)
$f(x)$ is a line
185.
(a)

Using the left rectangle rule, a computer takes two seconds to compute a particular definite integral accurate to 4 digits to the right of the decimal point. How long (in years) does it take to get 8 digits correct using the left rectangle rule? How about 12 digits? 20 digits?

```
ANSWER ©
WORKED SOLUTION ©
```

(b)

Repeat part a but this time assume that the trapezoidal rule is being used throughout.

```
ANSWER © 
WORKED SOLUTION ¢+
```

186. 

Given that $\int_{0}^{\infty} e^{-x^{2}} d x=\frac{\sqrt{\pi}}{2}$, find $\int_{0}^{\infty} x^{2} e^{-x^{2}} d x$.
187.

A population, $P$, is said to be growing logistically if the time, $T$, taken for it to increase from $P_{1}$ to $P_{2}$ is given by

$$
T=\int_{P_{1}}^{P_{2}} \frac{k d P}{P(L-P)}
$$

()
where $k$ and $L$ are positive constants and $P_{1}<P_{2}<L$.
(a)

Calculate the time taken for the population to grow from $P_{1}=L / 4$ to $P_{2}=L / 2$.

```
ANSWER © 
```

(b)

What happens to $T$ as $P_{2} \rightarrow L$ ?

## ANSWER $\oplus$

188. 

In 2011, the average per-capita income in the US was $\$ 41,560$ and increasing at a rate of $r(t)=$ $1122.12 e^{0.027 t}$ dollars per year, where $t$ is the number of years since 2011.
(a)

Estimate the average per-capita income in 2015.
(b)

Find a formula for the average per-capita income as a function of time after 2011.
189.

A patient is given an injection of Imitrex, a migraine medicine, at a rate of $r(t)=2 t e^{-2 t} \mathrm{ml} / \mathrm{sec}$, where $t$ is the number of seconds since the injection started.
(a)

By letting $t \rightarrow \infty$, estimate the total quantity of Imitrex injected.

## ANSWER ${ }^{+}$

WORKED SOLUTION ${ }^{+}$
(b)

What fraction of this dose has the patient received at the end of 5 seconds?

```
ANSWER © 
WORKED SOLUTION © 
```

190. 

In 1990 humans generated $1.4 \cdot 10^{20}$ joules of energy from petroleum. At the time, it was estimated that all of the earth's petroleum would generate approximately $10^{22}$ joules. Assuming the use of energy generated by petroleum increases by $2 \%$ each year, how long will it be before all of our petroleum resources are used up?

## 191.

An organism has a development time of $T$ days at a temperature $H=f(t)^{\circ} \mathrm{C}$. The total the number of degreedays $S$ required for development to maturity is a constant defined by

$$
S=\int_{0}^{T}\left(f(t)-H_{\min }\right) d t
$$

()
(a)

Evaluate this integral for $T=18, f(t)=30^{\circ} \mathrm{C}$, and $H_{\min }=10^{\circ} \mathrm{C}$. What are the units of $S$ ?

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ANSWER © 
```

(b)

Illustrate this definite integral on a graph. Label the features corresponding to $T, f(t), H_{\mathrm{min}}$, and $S$.
(c)

Now suppose $H=g(t)=20+10 \cos (2 \pi t / 6)^{\circ} \mathrm{C}$. Assuming that $S$ remains constant, write a definite integral which determines the new development time, $T_{2}$. Sketch a graph illustrating this new integral. Judging from the graph, how does $T_{2}$ compare to $T$ ? Find $T_{2}$.

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ANSWER © 
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192. 

For a positive integer $n$, let $\Psi_{n}(x)=C_{n} \sin (n \pi x)$ be the wave function used in describing the behavior of an electron. If $n$ and $m$ are different positive integers, find

$$
\int_{0}^{1} \Psi_{n}(x) \cdot \Psi_{m}(x) d x
$$

()

## CAS Challenge Problems

193. 

(a)

Use a computer algebra system to find $\int \frac{\ln x}{x} d x$,

$$
\int \frac{(\ln x)^{2}}{x} d x, \text { and } \int \frac{(\ln x)^{3}}{x} d x
$$

()

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```

(b)

Guess a formula for $\int \frac{(\ln x)^{n}}{x} d x$ that works for any positive integer $n$.

## ANSWER ©

WORKED SOLUTION $\oplus$
(c)

Use a substitution to check your formula.

## WORKED SOLUTION $\oplus$

194. 

(a)

Using a computer algebra system, find $\int(\ln x)^{n} d x$ for $n=1,2,3,4$.
(b)

There is a formula relating $\int(\ln x)^{n} d x$ to $\int(\ln x)^{n-1} d x$ for any positive integer $n$. Guess this formula using your answer to part a. Check your guess using integration by parts.

- In Problems 195-197:
- (a)Use a computer algebra system to find the indefinite integral of the given function.
- (b)Use the computer algebra system again to differentiate the result of part (a). Do not simplify.
- (c)Use algebra to show that the result of part (b) is the same as the original function. Show all the steps in your calculation.

195. 

$\sin ^{3} x$

## ANSWER $\oplus$

196. 

$\sin x \cos x \cos (2 x)$
197.

$$
\frac{x^{4}}{\left(1+x^{2}\right)^{2}}
$$

```
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```

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