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REVIEW EXERCISES AND PROBLEMS FOR CHAPTER 6

EXERCISES

1.

The graph of a derivative $f'(x)$ is shown in Figure 6.38. Fill in the table of values for $f(x)$ given that $f(0) = 2$.

x	0	1	2	3	4	5	6
$f(x)$	2						

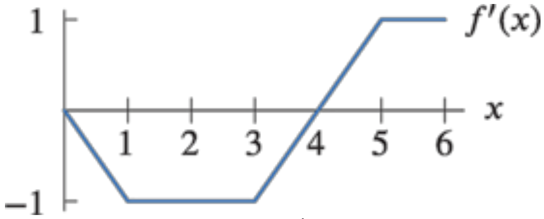


Figure 6.38: Graph of f', not f

WORKED SOLUTION

2.

Figure 6.39 shows f. If $F' = f$ and $F(0) = 0$, find $F(b)$ for $b = 1, 2, 3, 4, 5, 6$.

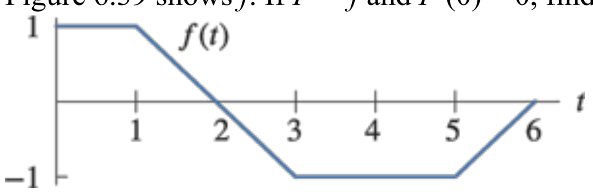
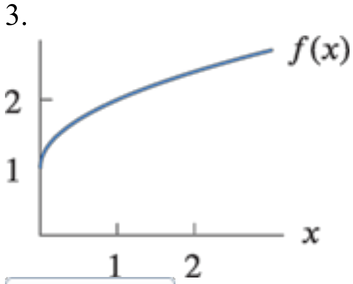


Figure 6.39

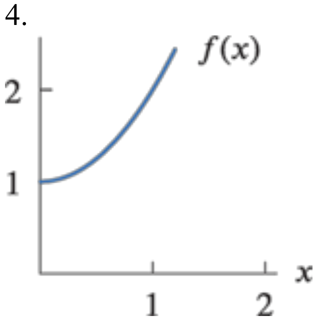
In Exercises 3-4, graph $F(x)$ such that $F'(x) = f(x)$ and $F(0) = 0$.

3.



ANSWER

4.



5.

(a)

Using Figure 6.40, estimate $\int_0^7 f(x)\,dx$.

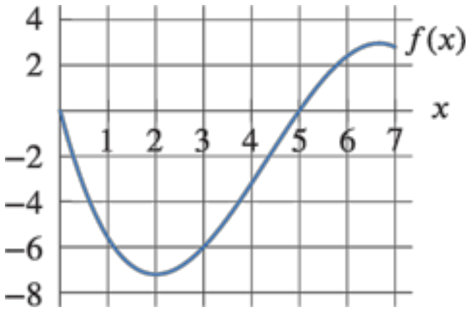


Figure 6.40

ANSWER

WORKED SOLUTION

(b)

If F is an antiderivative of the same function f and $F(0) = 25$, estimate $F(7)$.

ANSWER

WORKED SOLUTION

In Exercises 6-27, find the indefinite integrals.

6.

$\int 5x\,dx$

7.

$\int x^3\,dx$

ANSWER

8.

$\int \sin\theta\,d\theta$

9.

$\int (x^3 - 2)\,dx$

ANSWER

WORKED SOLUTION

10.

$$\int \left(t^2 + \frac{1}{t^2}\right) dt$$

11.

$$\int \frac{4}{t^2}\,dt$$

ANSWER ⊕

12.
$\int (x^2 + 5x + 8)\, dx$

13.

$$\int 4\sqrt{w}dw$$

ANSWER ⊕
WORKED SOLUTION ⊕

14.
$\int (4t + 7)\, dt$

15.
$\int \cos\theta\, d\theta$

ANSWER ⊕

16.

$$\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}}\right) dt$$

17.

$$\int \left(x + \frac{1}{\sqrt{x}}\right) dx$$

ANSWER ⊕
WORKED SOLUTION ⊕

18.

$$\int \left(\pi + x^{11}\right) dx$$

19.

$$\int \left(3\cos t + 3\sqrt{t}\right) dt$$

ANSWER ⊕

20.

$$\int \left(\frac{y^2 - 1}{y}\right)^2 dy$$

21.

$$\int \frac{1}{\cos^2 x}\, dx$$

ANSWER ⊕

WORKED SOLUTION ⊕

22.

$$\int \left(\frac{2}{x} + \pi \sin x\right) dx$$

23.

$$\int \left(\frac{x^2 + x + 1}{x}\right) dx$$

ANSWER ⊕

24.
$\int 5e^z\, dz$

25.
$\int 2^x\, dx$

ANSWER ⊕

WORKED SOLUTION ⊕

26.
$\int (3 \cos x - 7 \sin x)\, dx$

27.
$\int (2e^x - 8 \cos x)\, dx$

ANSWER ⊕

In Exercises 28-29, evaluate the definite integral exactly [as in $\ln (3\pi)$], using the Fundamental Theorem, and numerically [$\ln (3\pi) \approx 2.243$]:
28.

$$\int_{-3}^{-1} \frac{2}{r^3}\, d\pi$$

29.

$$\int_{-\pi/2}^{\pi/2} 2\cos\phi \, d\phi$$

ANSWER

WORKED SOLUTION

For Exercises 30-35, find an antiderivative $F(x)$ with $F'(x) = f(x)$ and $F(0) = 4$.

30.
$f(x) = x^2$

31.
$f(x) = x^3 + 6x^2 - 4$

ANSWER

32.

$$f(x) = \sqrt{x}$$

33.
$f(x) = e^x$

ANSWER

WORKED SOLUTION

34.
$f(x) = \sin x$

35.
$f(x) = \cos x$

ANSWER

36.
Show that $y = x + \sin x - \pi$ satisfies the initial value problem

$$\frac{dy}{dx} = 1 + \cos x, \quad y(\pi) = 0.$$

()

37.
Show that $y = x^n + A$ is a solution of the differential equation $y' = nx^{n-1}$ for any value of A.

WORKED SOLUTION

In Exercises 38-41, find the general solution of the differential equation.

38.

$$\frac{dy}{dx} = x^3 + 5$$

39.

$$\frac{dy}{dx} = 8x + \frac{1}{x}$$

ANSWER ⊕

40.

$$\frac{dW}{dt} = 4\sqrt{t}$$

41.

$$\frac{dr}{dp} = 3 \sin p$$

ANSWER ⊕

WORKED SOLUTION ⊕

In Exercises 42-45, find the solution of the initial value problem.

42.

$$\frac{dy}{dx} = 6x^2 + 4x, \quad y(2) = 10$$

43.

$$\frac{dP}{dt} = 10e^t, \quad P(0) = 25$$

ANSWER ⊕

44.

$$\frac{ds}{dt} = -32t + 100, \quad s = 50 \text{ when } t = 0$$

45.

$$\frac{dq}{dz} = 2 + \sin z, \quad q = 5 \text{ when } z = 0$$

ANSWER ⊕

WORKED SOLUTION ⊕

Find the derivatives in Exercises 46-47.

46.

$$\frac{d}{dt}\int_t^{\pi} \cos\left(z^3\right) dz$$

47.

$$\frac{d}{dx}\int_x^{1} \ln t \, dt$$

ANSWER

PROBLEMS

48.

Using the graph of g' in Figure 6.41 and the fact that $g(0) = 50$, sketch the graph of $g(x)$. Give the coordinates of all critical points and inflection points of g.

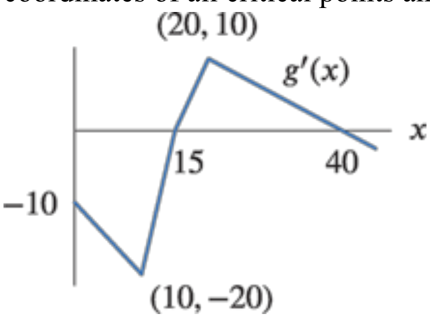


Figure 6.41

49.

The vertical velocity of a cork bobbing up and down on the waves in the sea is given by Figure 6.42. Upward is considered positive. Describe the motion of the cork at each of the labeled points. At which point(s), if any, is the acceleration zero? Sketch a graph of the height of the cork above the sea floor as a function of time.

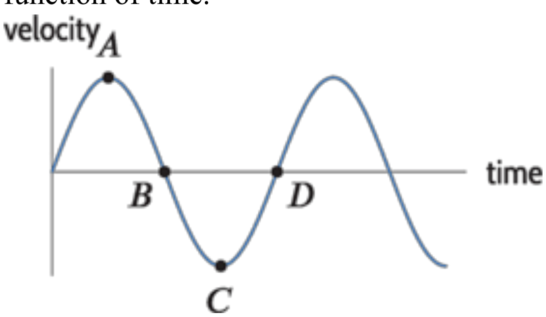


Figure 6.42

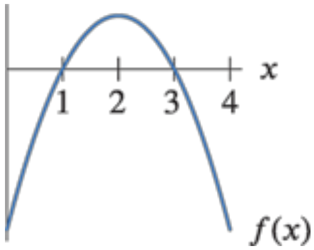
ANSWER

WORKED SOLUTION

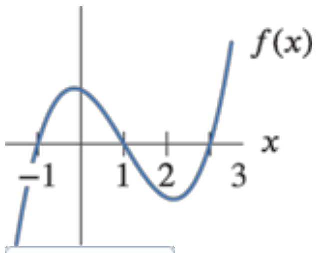
In Problems 50-51, a graph of f is given. Let $F'(x) = f(x)$.

- (a)What are the x-coordinates of the critical points of $F(x)$?
- (b)Which critical points are local maxima, which are local minima, and which are neither?
- (c)Sketch a possible graph of $F(x)$.

50.



51.



ANSWER

52.

The graph of f'' is given in Figure 6.43. Draw graphs of f and f', assuming both go through the origin, and use them to decide at which of the labeled x-values:

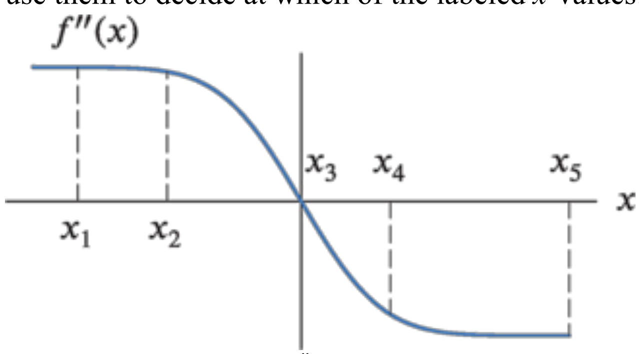


Figure 6.43: Graph of f''

(a)

$f(x)$ is greatest.

(b)

$f(x)$ is least.

(c)

$f'(x)$ is greatest.

(d)

$f'(x)$ is least.

(e)

$f''(x)$ is greatest.

(f)

$f''(x)$ is least.

53.

Assume f' is given by the graph in Figure 6.44. Suppose f is continuous and that $f(3) = 0$.

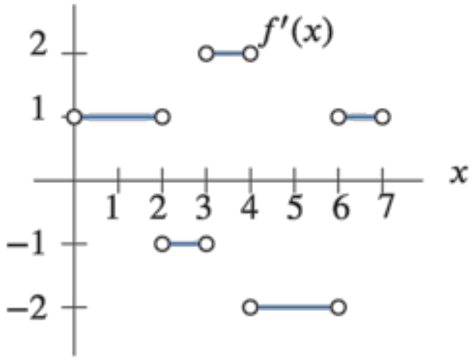


Figure 6.44

(a)

Sketch a graph of f.

ANSWER

WORKED SOLUTION

(b)

Find $f(0)$ and $f(7)$.

ANSWER

WORKED SOLUTION

(c)

Find $\int_0^7 f'(x)\,dx$ in two different ways.

ANSWER

WORKED SOLUTION

54.

Use the Fundamental Theorem to find the area under $f(x) = x^2$ between $x = 1$ and $x = 4$.

55.

Calculate the exact area between the x-axis and the graph of $y = 7 - 8x + x^2$.

ANSWER

56.

Find the exact area below the curve $y = x^3(1 - x)$ and above the x-axis.

57.

Find the exact area enclosed by the curve $y = x^2(1 - x)$ and the x-axis.

ANSWER

WORKED SOLUTION

58.

Find the exact area between the curves $y = x^2$ and $x = y^2$.

59.
Calculate the exact area above the graph of $y = \sin\theta$ and below the graph of $y = \cos\theta$ for $0 \le \theta \le \pi/4$.
ANSWER ⊕

60.
Find the exact area between $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ for $0 \le \theta \le 2\pi$.

61.
Find the exact value of the area between the graphs of $y = \cos x$ and $y = e^x$ for $0 \le x \le 1$.
ANSWER ⊕
WORKED SOLUTION ⊕

62.
Find the exact value of the area between the graphs of $y = \sinh x$, $y = \cosh x$, for $-1 \le x \le 1$.

63.
Find the exact positive value of c if the area between the graph of $y = x^2 - c^2$ and the x-axis is 36.
ANSWER ⊕

64.
Use the Fundamental Theorem to find the average value of $f(x) = x^2 + 1$ on the interval $x = 0$ to $x = 10$. Illustrate your answer on a graph of $f(x)$.

65.
The average value of the function $v(x) = 6/x^2$ on the interval $[1, c]$ is equal to 1. Find the value of c.
ANSWER ⊕
WORKED SOLUTION ⊕

In Problems 66-68, evaluate the expression using $f(x) = 5\sqrt{x}$.

66.

$$\int_1^4 f^{-1}(x)\,dx$$

67.

$$\int_1^4 (f(x))^{-1}\,dx$$

ANSWER ⊕

68.

$$\left(\int_1^4 f(x)\,dx\right)^{-1}$$

In Problems 69-70, evaluate and simplify the expressions given that $f(t) = \int_0^t tx^2\,dx$.

69.
$f(2)$

ANSWER

WORKED SOLUTION

70.
$f(n)$

Calculate the derivatives in Problems 71-74.

71.

$$\frac{d}{dx}\int_{2}^{x^3} \sin\left(t^2\right) dt$$

ANSWER

72.

$$\frac{d}{dx}\int_{\cos x}^{3} e^{t^2}\, dt$$

73.

$$\frac{d}{dx}\int_{-x}^{x} e^{-t^4}\, dt$$

ANSWER

WORKED SOLUTION

74.

$$\frac{d}{dt}\int_{e^t}^{t^3} \sqrt{1+x^2}\,dx$$

75.
A store has an inventory of Q units of a product at time $t = 0$. The store sells the product at the steady rate of Q/A units per week, and it exhausts the inventory in A weeks.

(a)
Find a formula $f(t)$ for the amount of product in inventory at time t. Graph $f(t)$.

ANSWER

(b)
Find the average inventory level during the period $0 \le t \le A$. Explain why your answer is reasonable.

ANSWER

76.
For $0 \le t \le 10$ seconds, a car moves along a straight line with velocity

$$v(t) = 2 + 10t \text{ ft/sec.}$$

()

(a)

Graph $v(t)$ and find the total distance the car has traveled between $t = 0$ and $t = 10$ seconds using the formula for the area of a trapezoid.

(b)

Find the function $s(t)$ that gives the position of the car as a function of time. Explain the meaning of any new constants.

(c)

Use your function $s(t)$ to find the total distance traveled by the car between $t = 0$ and $t = 10$ seconds. Compare with your answer in part a.

(d)

Explain how your answers to parts a and c relate to the Fundamental Theorem of Calculus.

77.

For a function f, you are given the graph of the derivative f' in Figure 6.45 and that $f(0) = 50$.

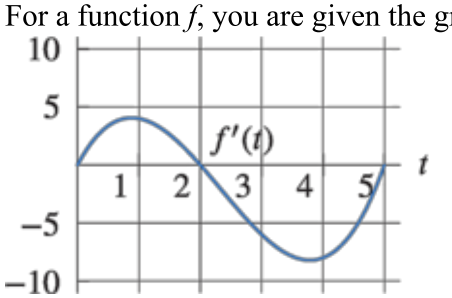


Figure 6.45

(a)

On the interval $0 \le t \le 5$, at what value of t does f appear to reach its maximum value? Its minimum value?

ANSWER

WORKED SOLUTION

(b)

Estimate these maximum and minimum values.

ANSWER

WORKED SOLUTION

(c)

Estimate $f(5) - f(0)$.

ANSWER

WORKED SOLUTION

78.

The acceleration, a, of a particle as a function of time is shown in Figure 6.46. Sketch graphs of velocity and position against time. The particle starts at rest at the origin.

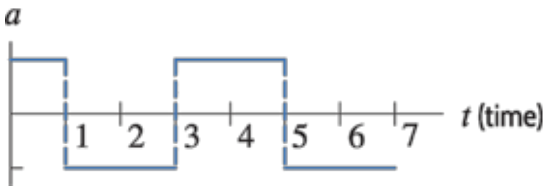


Figure 6.46

79.

The angular speed of a car engine increases from 1100 revs/min to 2500 revs/min in 6 sec.

(a)

Assuming that it is constant, find the angular acceleration in revs/min^2.

ANSWER

(b)

How many revolutions does the engine make in this time?

ANSWER

80.

Figure 6.47 is a graph of

$$f(x) = \begin{cases} -x + 1, & \text{for } 0 \le x \le 1; \\ x - 1, & \text{for } 1 < x \le 2. \end{cases}$$

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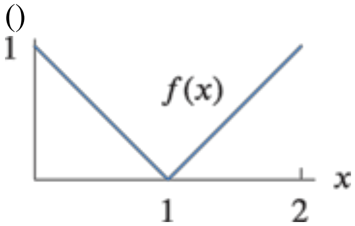

Figure 6.47

(a)

Find a function F such that $F' = f$ and $F(1) = 1$.

(b)

Use geometry to show the area under the graph of f above the x-axis between $x = 0$ and $x = 2$ is equal to $F(2) - F(0)$.

(c)

Use parts a and b to check the Fundamental Theorem of Calculus.

81.

If a car goes from 0 to 80 mph in six seconds with constant acceleration, what is that acceleration?

ANSWER

WORKED SOLUTION

82.

A car going at 30 ft/sec decelerates at a constant 5 ft/sec^2.

(a)

Draw up a table showing the velocity of the car every half second. When does the car come to rest?

(b)

Using your table, find left and right sums which estimate the total distance traveled before the car comes to rest. Which is an overestimate, and which is an underestimate?

(c)

Sketch a graph of velocity against time. On the graph, show an area representing the distance traveled before the car comes to rest. Use the graph to calculate this distance.

(d)

Now find a formula for the velocity of the car as a function of time, and then find the total distance traveled by antidifferentiation. What is the relationship between your answer to parts c and d and your estimates in part b?

83.

An object is thrown vertically upward with a velocity of 80 ft/sec.

(a)

Make a table showing its velocity every second.

ANSWER ⊕

(b)

When does it reach its highest point? When does it hit the ground?

ANSWER ⊕

(c)

Using your table, write left and right sums which under- and overestimate the height the object attains.

ANSWER ⊕

(d)

Use antidifferentiation to find the greatest height it reaches.

ANSWER ⊕

84.

If $A(r)$ represents the area of a circle of radius r and $C(r)$ represents its circumference, it can be shown that $A'(r) = C(r)$. Use the fact that $C(r) = 2\pi r$ to obtain the formula for $A(r)$.

85.

If $V(r)$ represents the volume of a sphere of radius r and $S(r)$ represents its surface area, it can be shown that $V'(r) = S(r)$. Use the fact that $S(r) = 4\pi r^2$ to obtain the formula for $V(r)$.

ANSWER ⊕

WORKED SOLUTION ⊕

86.

A car, initially moving at 60 mph, has a constant deceleration and stops in a distance of 200 feet. What is its deceleration? (Give your answer in ft/sec^2. Note that 1 mph = 22/15 ft/sec.)

87.
Water flows at a constant rate into the left side of the W-shaped container in Figure 6.48. Sketch a graph of the height, H, of the water in the left side of the container as a function of time, t. The container starts empty.

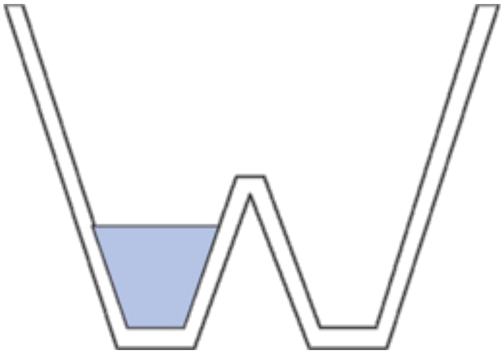

Figure 6.48

ANSWER ⊕

88.
(a)
Explain why you can rewrite x^x as $x^x = e^{x \ln x}$ for $x > 0$.

(b)
Use your answer to part a to find $\frac{d}{dx}\left(x^x\right)$.

(c)
Find $\int x^x (1 + \ln x)\, dx$.

(d)
Find $\int_1^2 x^x (1 + \ln x)\, dx$ exactly using part c. Check your answer numerically.

In Problems 89-90, the quantity, $N(t)$ in kg, of pollutant that has leeched from a toxic waste site after t days, is given by

$$N(t) = \int_0^t r(x)\, dx, \quad \text{where } r(x) > 0, r'(x) < 0.$$

()
89.
If there is enough information to decide, determine whether $N(t)$ is an increasing or a decreasing function and whether its graph concave up or concave down.

ANSWER ⊕

WORKED SOLUTION ⊕

90.

Rank in order from least to greatest:

$$N(20), N(10), N(20) - N(10), N(15) - N(5).$$

()

91.
Let $f(x)$ have one zero, at $x = 3$, and suppose $f'(x) < 0$ for all x and that

$$\int_0^3 f(t)\,dt = -\int_3^5 f(t)\,dt.$$

()
Define $F(x) = \int_0^x f(t)\,dt$ and $G(x) = \int_1^x F(t)\,dt$.
(a)
Find the zeros and critical points of F.
ANSWER

(b)
Find the zeros and critical points of G.
ANSWER

92.
Let $P(x)\int_0^x \arctan\left(t^2\right)\,dt$.
(a)
Evaluate $P(0)$ and determine if P is an even or an odd function.

(b)
Is P increasing or decreasing?

(c)
What can you say about concavity?

(d)
Sketch a graph of $P(x)$.

CAS Challenge Problems

93.
(a)
Set up a right-hand Riemann sum for $\int_a^b x^3\,dx$ using n subdivisions. What is Δx? Express each x_i, for $i = 1, 2, \ldots, n$, in terms of i.
ANSWER
WORKED SOLUTION

(b)
Use a computer algebra system to find an expression for the Riemann sum in part a; then find the limit of this expression as $n \to \infty$.

ANSWER

WORKED SOLUTION

(c)
Simplify the final expression and compare the result to that obtained using the Fundamental Theorem of Calculus.

WORKED SOLUTION

94.
(a)
Use a computer algebra system to find $\int e^{2x}\,dx$, $\int e^{3x}\,dx$, and $\int e^{3x+5}\,dx$.

(b)
Using your answers to part a, conjecture a formula for $\int e^{ax+b}\,dx$, where a and b are constants.

(c)
Check your formula by differentiation. Explain which differentiation rules you are using.

95.
(a)
Use a computer algebra system to find $\int \sin(3x)\,dx$, $\int \sin(4x)\,dx$, and $\int \sin(3x-2)\,dx$.

ANSWER

(b)
Using your answers to part a, conjecture a formula for $\int \sin(ax+b)\,dx$, where a and b are constants.

ANSWER

(c)
Check your formula by differentiation. Explain which differentiation rules you are using.

96.
(a)
Use a computer algebra system to find

$$\int \frac{x-2}{x-1}\,dx,\ \int \frac{x-3}{x-1}\,dx,\text{ and }\int \frac{x-1}{x-2}\,dx.$$

()

(b)
If a and b are constants, use your answers to part a to conjecture a formula for

$$\int \frac{x-a}{x-b}\,dx.$$

()

(c)
Check your formula by differentiation. Explain which rules of differentiation you are using.

97.
(a)
Use a computer algebra system to find

$$\int \frac{1}{(x-1)(x-3)}\,dx, \int \frac{1}{(x-1)(x-4)}\,dx$$

()
and

$$\int \frac{1}{(x-1)(x+3)}\,dx.$$

()
ANSWER
WORKED SOLUTION

(b)
If a and b are constants, use your answers to part a to conjecture a formula for

$$\int \frac{1}{(x-a)(x-b)}\,dx.$$

()
ANSWER
WORKED SOLUTION

(c)
Check your formula by differentiation. Explain which rules of differentiation you are using.
WORKED SOLUTION