Differential Equations Name: **Solutions**

Catenary Problems

1. A 625 foot wire weighing 2 lb.s per foot is suspended between two towers at the same level. The sag is 25 feet.

(a) Determine the equation of this particular catenary.

Solution: We have two pieces of information to use, namely the arclength and the sag. From the notes we know that $2c \cdot \sinh\left(\frac{a}{2c}\right) = 625$. The sag is the measure of the difference between the lowest and highest points on the curve so $d = c \cdot \cosh\frac{a}{2c} - c$ and therefore we have the system of equations:

$$2c \cdot \sinh\left(\frac{a}{2c}\right) = 625 \quad (1)$$

$$c \cdot \cosh\left(\frac{a}{2c}\right) - c = 25 \quad (2)$$

We want to make use of the relationship $\cosh^2 x - \sinh^2 x = 1$ so by solving for $\sinh\frac{a}{2c}$ and $\cosh\frac{a}{2c}$ respectively and squaring, we have:

$$c^2 \cdot \sinh^2\left(\frac{a}{2c}\right) = \left(\frac{625}{2}\right)^2 \quad (3)$$

$$c^2 \cdot \cosh^2\left(\frac{a}{2c}\right) = (25+c)^2 \quad (4)$$

Then subtracting (3) from (4) we get $c^2 = (25+c)^2 - \left(\frac{625}{2}\right)^2$ and solving gives us $c = \frac{15525}{8} = 1940.625$
It follows that $y = 1940.625\cosh\left(\frac{x}{3881.25}\right)$

□

(b) How far apart are the towers?

Solution: This is the continuation of the previous problem where we had both a and c as unknowns. From the arclength equation we know $2c \cdot \sinh\frac{a}{2c} = 625$ and since $c = 1940.625$, it follows that $3881.25\sinh\left(\frac{a}{3881.25}\right) = 625$. Solving gives us $\sinh\left(\frac{a}{3881.25}\right) \approx 0.161$, so $a = 3881.25\sinh^{-1}(0.161) \approx 622.33$. The towers are about $622'$ apart.

□

(c) Determine the maximum tension in the wire.

Solution: From the notes we see the tension is given by $T = \delta\sqrt{(l/2)^2 + c^2}$ and since we have both $l = 625$ and $c = 1940.625$, it follows that $T = 2\sqrt{(312.5)^2 + (1940.625)^2} \approx 3931.25$
So the maximum tension is 3931.25 pounds.

□

(d) Determine the angle θ that corresponds to the maximum tension.

Solution: From the discussion in the text, we see that $\sin\theta = \frac{W}{T}$, where W is the weight of the section between the low point, B, and the point $A(x, y)$. Since the maximum tension occurs at the endpoint of the catenary, we have $W = 2 \cdot l/2 = 625$ lb.s. Then $\theta = \arcsin(625/3931.25) \approx 0.16$ radians or about $9°$.

□