Quiz 7.3 Name:____________________________

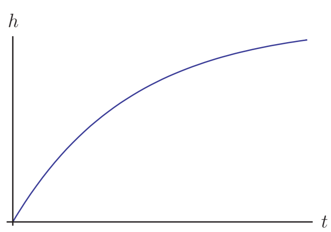
Show all relevant work!

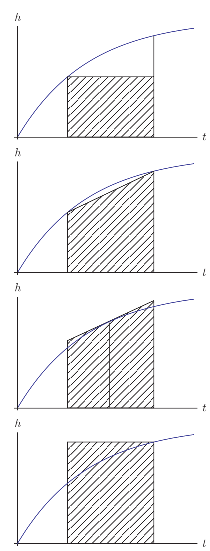
1. The rate, r, at which people get sick during an epidemic of the flu can be approximated by $r(t) = 1000te^{-0.05t}$, where r is measured in people/day and t is measured in days since the start of the epidemic. Projecting into the indefinite future, what is the total number of people who get sick from this strain of the flu? (Find any integrals using the FTC.)

$$1000\int_0^\infty te^{-0.05t}\,\mathrm{d}t \to \begin{cases} u = t & : \quad v' = e^{-0.05t} \\ u' = 1 & : \quad v = -20e^{-0.05t} \end{cases}$$

$$\begin{aligned} 1000\int_0^\infty te^{-0.05t}\,\mathrm{d}t &= 1000\left[-20te^{-0.05t} + 20\int_0^\infty e^{-0.05t}\,\mathrm{d}t\right]\Bigg|_0^\infty \\ &= 1000\left[-20te^{-0.05t} - 400e^{-0.05t}\right]\Big|_0^\infty \\ &= -20000\lim_{t\to\infty}\frac{t+20}{e^{0.05t}} + 20000\cdot\frac{(0+20)}{e^{0.05(0)}} \\ &= -20000\lim_{t\to\infty}\frac{t+20}{e^{0.05t}} \overset{L'H}{\longrightarrow} \lim_{t\to\infty}\frac{1}{0.05e^{0.05t}} \to 0 + 400000 \\ &= 400,000 \text{ people.} \end{aligned}$$

2. The graph of $f(x)$ is shown below. The results from LHS(n), RHS(n), TRAP(n), and MID(n) used to approximate $\int_0^3 f(x)\,\mathrm{d}x$ (with the same n for each) are 7.057, 7.519, 7.656, and 7.981 (no particular order). Match each rule with its approximation and determine SIMP(n) for this same n.





LHS $\longrightarrow$ 7.057

TRAP $\longrightarrow$ 7.519

MID $\longrightarrow$ 7.656

RHS $\longrightarrow$ 7.981

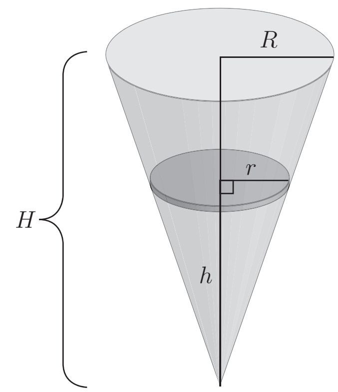
$$\text{SIMP}(n) = \frac{2(7.656) + 7.519}{3} = 7.610$$

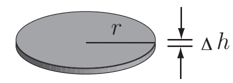
3. Determine whether $\displaystyle\int_1^\infty \frac{\sin x + 2}{\sqrt{x^3+1}}\,\mathrm{d}x$ converges or diverges using the comparison test.

Consider the numerator first. Since $-1 \le \sin x \le 1$ we know that $1 \le \sin x + 2 \le 3$.
Now consider the denominator. We have something of the form $\dfrac{k}{\sqrt{x^3+1}}$ Eliminating the $+1$ would give the simpler expression $\dfrac{k}{\sqrt{x^3}} = \dfrac{k}{x^{3/2}}$ and . Since we know $\displaystyle\int_1^\infty \frac{1}{x^p}\,\mathrm{d}x$ converges for $p > 1$, we know $k\displaystyle\int_1^\infty \frac{1}{x^{3/2}}\,\mathrm{d}x$ converges. Choosing $k = 3$ guarantees $\dfrac{3}{x^{3/2}} > \dfrac{k}{\sqrt{x^3+1}}$ and by the comparison test, $\displaystyle\int_1^\infty \frac{\sin x + 2}{\sqrt{x^3+1}}\,\mathrm{d}x$ converges.

4. Find, by slicing, a formula for the volume of a cone of height, H, and base radius, R.

Taking slices parallel to the base gives circles of radius, r, at a distance h from the bottom (peak) of the cone.





From the diagram we have $\frac{r}{h} = \frac{R}{H} \longrightarrow r = \frac{R}{H}h$

It follows that $\Delta V = \pi r^2 \Delta h = \pi \left(\frac{R}{H}h\right)^2 \Delta h$

We generalize from one slice to the sum of infinitely many slices of infinitessimally small thickness and write:

$$\begin{aligned} V &= \int_0^H \pi \frac{R^2}{H^2} h^2 \, \mathrm{d}h && (1) \\ &= \pi \frac{R^2}{H^2} \int_0^H h^2 \, \mathrm{d}h && (2) \\ &= \pi \frac{R^2}{H^2} \cdot \frac{h^3}{3} \bigg|_0^H && (3) \\ &= \pi \frac{R^2}{H^2} \cdot \frac{H^3}{3} && (4) \\ &= \frac{1}{3} \pi R^2 H && (5) \end{aligned}$$