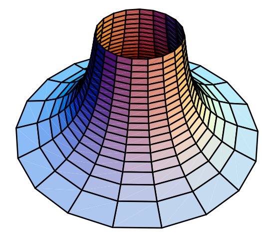
Honors Project #2 Due Date:____________________

Houdini's Escape[1]

Harry Houdini was a famous escape artist. In this project we will relive a trick of his that challenged his mathematical prowess, as well as his skill and bravery. It will challenge these qualities in you as well.

Houdini had his feet shackled to the top of a concrete block which was placed on the bottom of a giant laboratory flask. The cross sectional radius of the flask, measured in feet, was given as a function of the height z from the ground by the formula:



$$r(z) = \frac{10}{\sqrt{z}}$$

with the bottom of the flask at $z = 1$ foot. The flask was then filled with water at a steady rate of 22π cubic feet per minute. Houdini's job was to escape the shackles before he was drowned by the rising water in the flask.

Now Houdini knew it would take him exactly ten minutes to escape the shackles. For dramatic impact, he wanted to time his escape so it was completed precisely at the moment the water level reached the top of his head. Houdini was exactly six feet tall. In the design of the apparatus he was allowed to specify only one thing, the height of the concrete block he stood on.

Problems to Solve:

A. Your task is to find out how high this block should be. Express the volume of the water in the flask as a function of the height of the liquid above ground level. What is the volume when the water level reaches the top of Houdini's head? (Neglect Houdini's volume and the volume of the block.) What is the height of the block?

B. Let $h(t)$ be the height of the water above ground level at time t. In order to check the progress of his escape moment by moment, Houdini derives the equation for the rate of change $\frac{\mathrm{d}h}{\mathrm{d}t}$ as a function of $h(t)$ itself. Derive this equation. How fast is the water level changing when the flask starts to fill? How fast is it changing when the water reaches the top of his head? Express $h(t)$ as a function of time.

C. Houdini would like to be able to perform this trick with any flask. Help him plan his next trick by generalizing the derivation of part (b). Consider a flask with cross sectional radius $r(z)$ (an arbitrary function of z) and a constant inflow rate $\frac{\mathrm{d}V(t)}{\mathrm{d}t} = A$. Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ as a function of $h(t)$.

Completed Project:

When you have done the work necessary to complete the project, you need to prepare it in written form. The paper you submit should have a mix of equations, formulas, diagrams, and prose to support your conclusions. Use complete sentences, good grammar, and correct punctuation. The prose should be written in order to convey to the reader an explanation of what you have done. It should be written in such a way that it can be read and understood by anyone familiar with the material in this course. You will be graded on your written presentation as well as the mathematical content. Be sure to include all of your reasoning and cite an resources you used in finding your solution.

[1]Reproduced from Cohen, Gaughan, et al. *Student Research Projects in Calculus*;MAA: Washington, D.C.,1991