Velocity is a vector measuring the rate at which an object's position changes with time. It gives both direction (through its sign) and magnitude in the size of the value associated with it. The magnitude of velocity is what we call *speed* and is typically written, speed $= |v|$. Therefore, when we speak about speed we ignore the direction an object is moving in and focus only on how quickly its distance is changing with time.

Similarly, displacement is a vector, concerned with location relative to a particular starting point, while distance is concerned not with the position of the end result but with the ground traveled in getting there.

The problems below give some opportunities to develop your insight with velocity and speed.

$\boxed{1.}$ The velocity of an African Swallow (in feet per second) as it flies north–south, is shown in the graph below. Use the graph to answer the following questions.

(a) What is the maximum velocity of the bird on this interval?

Solution: Maximum positive value for $v(t)$ is $v = 30$ (at $t = 20$ sec.).

(b) What is the maximum speed of the bird on this interval?

Solution: Since speed is the magnitude of velocity, $|v|$, we choose the greatest deviation from 0 at $v = -50$, so max speed is 50 mph (south).
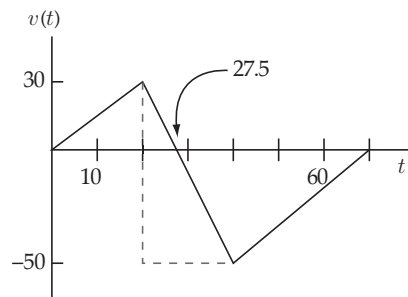
(c) How has the Swallow's position changed over the first 70 seconds relative to its starting position? (Think displacement)

Solution: Since position depends on direction, we need to distinguish between positive and negative velocity. From the graph, we can make similar triangles to find $\frac{30}{x} = \frac{80}{20} \longrightarrow x = 7.5$ so the t-intercept is at $20 + 7.5 = 27.5$ sec.

The distance traveled north is therefore $\frac{1}{2}(27.5)(30) = 412.5$ ft. The distance traveled south is $\frac{1}{2}(42.5)(50) = 1062.5$ ft. Therefore the net displacement of the Swallow, is $412.5 - 1062.5 = -650$ or 650 feet south of where it began.

(d) How far has the bird traveled over the first 70 seconds?

Solution: From above we add the two distances since we're only interested in ground covered, not relative position: $412.5 + 1062.5 = 1475$ ft.

(e) What is the average velocity of the bird over this interval?

Solution: From (c), -650 ft/70 sec ≈ -9.3 ft/sec.

(f) What is the average speed of the bird over this interval?

**Solution:** From (d), 1475 ft/70 sec ≈ 21.1 ft/sec.

$\boxed{2.}$ The velocity of a bicycle (in miles per hour) as it rides east–west, is shown in the graph below. Use the graph to answer the following questions.

(a) What is the average velocity of the bike over the first 6 hours?

Solution: 0. (Equal but opposite areas).

(b) What is the average speed of the bike over the first 6 hours?

Solution: From (c), $\left(\frac{1}{2}(20)(3+2) + \frac{1}{2}(20)(3+2)\right)/6 \approx 16.7$ ft/sec.
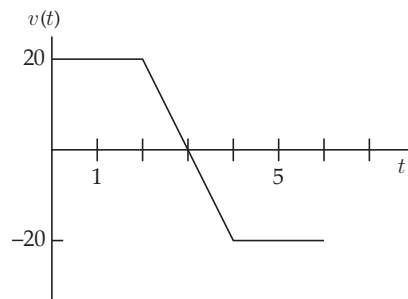
(c) Discuss the displacement of the bicycle versus the total distance it has traveled over the first 6 hours.

Solution: Displacement is 0 (didn't really get anywhere). Total distance is 100 miles (see part (c)).