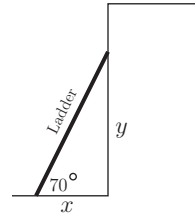


1. If a fireman leans a 24 foot ladder against a building at a 70° angle,

(a) How far from the building is the base of the ladder?

Solution: We want the distance from the building (labeled x). We know the length of the ladder is 24' so relative to the 70° angle we **know** the hypotenuse and we **want** the side adjacent so we use cosine:

$$\cos 70^\circ = \frac{x}{24} \longrightarrow 24 \cos 70^\circ \approx 8.2' \quad \square$$



(b) How high (above the ground) does the ladder reach on the building?

Solution: We want the distance from the building (labeled y). We know the length of the ladder is 24' so relative to the 70° angle we **know** the hypotenuse and we **want** the side opposite so we use sine:

$$\sin 70^\circ = \frac{y}{24} \longrightarrow 24 \sin 70^\circ \approx 22.6' \quad \square$$

2. A merry-go-round with a 10 ft. diameter is spinning at 12rpm.

(a) What is the angular velocity of the merry-go-round in radians per second?

Solution: Since there are 2π radians in one full revolution, the merry-go-round is rotating at $12 \cdot 2\pi = 24\pi$ radians per minute. Then this is the same as $24\pi \frac{\text{rad}}{\text{minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} = \frac{2\pi}{5}$ rad per second. \square

(b) How fast (in feet per second) is Raul traveling if he sits on the outer edge of the merry-go-round?

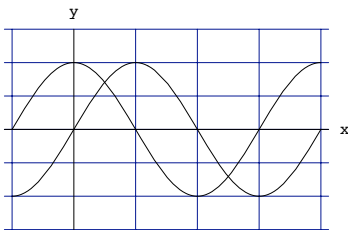
Solution: Since speed is measured in distance traveled over time, we need to know how far Raul travels in a second or minute. Since he makes 12 full revolutions in a minute and the radius of the merry-go-round is 5ft, he's covering $12 \cdot 2\pi(5) = 120\pi$ feet per minute or equivalently $\frac{120\pi}{60} = 2\pi$ ft/sec. \square

(c) How fast is Klaus travelling (in feet per second) if he sits at the center of the merry-go-round?

Solution: Since Klaus is at the center his distance from the center is 0 and therefore isn't going anywhere. While his angular velocity is $\frac{2\pi}{5}$ rad per second, his speed is 0. \square

3. The functions $\sin x$ and $\cos x$ are almost identical except for horizontal position (see below). That means you should be able to express $\sin x$ as a shift of $\cos x$ and similarly, $\cos x$ as a shift of $\sin x$.

Specifically, find c so that $\sin x = \cos(x + c)$ and find c so that $\cos x = \sin(x + c)$.



Solution: In both cases we note that cosine reaches its maximum at $x = 0$, that is $\cos(0) = 1$. Meanwhile sine doesn't reach its maximum until $\frac{\pi}{2}$ radians later. That means in order to get cosine shifted ahead to sine we have to shift it forward by $\frac{\pi}{2}$ radians. Therefore, $\sin x = \cos(x - \frac{\pi}{2})$.

Similarly, for sine to catch up with cosine, it has to be shifted ahead by $\frac{\pi}{2}$ radians so $\cos x = \sin(x + \frac{\pi}{2})$. \square

4. Solve the following for $x \in \mathbb{R}$.

(a) $\sin x = 1$

Solution: On the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (this is the interval we restrict sine to in order to make arcsin a function) we know $\sin x = 1$ when $x = \arcsin(1) = \frac{\pi}{2}$. Since this is the only occurrence on that interval and sine has a period of 2π , the general solution is $x = \frac{\pi}{2} + 2\pi n$, where $n \in \mathbb{Z}$. \square

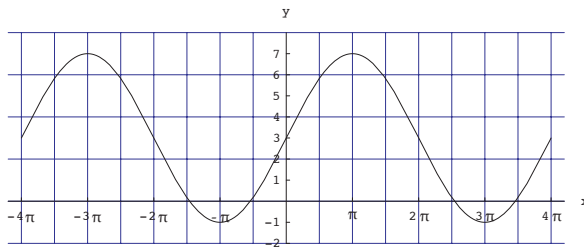
(b) $\cos 3x = 1$

Solution: Similar logic here, cosine is usually restricted to the interval $[0, \pi]$ in order to invert it. Then $3x = \arccos(1) \longrightarrow x = 0$. Since $\cos(3x)$ has a period of $\frac{2\pi}{3}$, the general solution is $x = 0 + \frac{2\pi}{3}n$ $n \in \mathbb{Z}$. \square

(c) $\sin(x^2 - 1) = 1$

Solution: From part (a) we know the solution to $\sin(\) = 1$ is $x = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$. In this case x is replaced with $x^2 - 1$ so we have $x^2 - 1 = \frac{\pi}{2} + 2\pi n$ ($n \in \mathbb{Z}$). Solving gives us $x = \pm\sqrt{1 + \frac{\pi}{2} + 2\pi n}$, ($n \in \mathbb{Z}_{\geq 0}$). Notice the change from $n \in \mathbb{Z}$ to n in the non-negative integers since the square root has a restricted domain. \square

5. Find two different equations for the periodic function shown on the right. (One in terms of $\sin x$ and the other in terms of $\cos x$).



Solution: We need to find the constants A, B, C and D in the form $y = A \sin(Bx + D) + C$
 $C = \frac{-1+7}{2} = 3$
 $A = 7 - 3 = 4$

The span from peak to peak is 4π (or the span from top to bottom is 2π) so the period is 4π and therefore $B = \frac{1}{2}$.

Finally, since the maximum occurs at π when for sine it normally occurs at $\frac{\pi}{2}$, we need to find C so that $\frac{1}{2}(\pi) + D = \pi \rightarrow D = 0$.

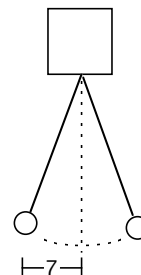
This gives us $y = 4 \sin(\frac{1}{2}x) + 3$ \square

Similarly, for cosine we have the same A, B and C so we only need to find D . The maximum of cosine occurs at 0 so we need D so that $\frac{1}{2}(\pi) + D = 0 \rightarrow D = -\frac{\pi}{2}$.

This gives us $y = 4 \cos(\frac{1}{2}x - \frac{\pi}{2}) + 3$ \square

6. A pendulum on a clock keeps time by swinging and clicking the second hand gear. The pendulum begins displaced 7 cm from center and swings to the other extreme in 1 second (and back in one second and so on). Write a periodic equation for the pendulum's displacement from center as a function of time.

Solution: Again, we need to find the constants A, B, C and D in the form $y = A \sin(Bx + D) + C$. The amplitude, A is the displacement from center so $A = 7$ if we make the center $C = 0$. The period is 2 seconds (to return to start) and therefore $B = \pi$. Finally, since the pendulum begins at its maximum displacement we can use cosine (or $-\cos$) and therefore $D = 0$. This gives us $d = 7 \cos(\pi t)$. \square



7. The table below shows the US average unemployment rate at the beginning of each year from 1993 – 2003. (Where 1993 is $t = 0$).

t (years)	0	1	2	3	4	5	6	7	8	9	10
$U(t)$ (% unemployment)	7.3	6.6	5.6	5.6	5.3	4.6	4.3	4.0	4.1	5.6	5.7

a) Write a periodic function that models these data using methods discussed in class.

Solution: The maximum unemployment is 7.3% while the minimum is 4.0% so $C = \frac{7.3+4}{2} = 5.65$.

Then $A = 7.3 - 5.65 = 1.65$

It takes seven years to go from the highest to the lowest value so 7 years is half a period and therefore the period is 14 years. It follows that $B = \frac{2\pi}{14} = \frac{\pi}{7}$.

Finally, if we use cosine which begins at its maximum like the data set, we can ignore D .

This gives us $U(t) = 1.65 \cos(\frac{\pi}{7}t) + 5.65$ \square

b) Use your model to predict the unemployment rate for January of 2006.

Solution: From the equation in (a) we have $U(13) = 1.65 \cos(\frac{\pi}{7}(13)) + 5.65 \approx 7.1\%$ unemployment. \square

(Note that your calculator needs to be in radians, not degrees).

The actual unemployment rate in 2006 was 4.7% which should give us pause about extrapolating from this model