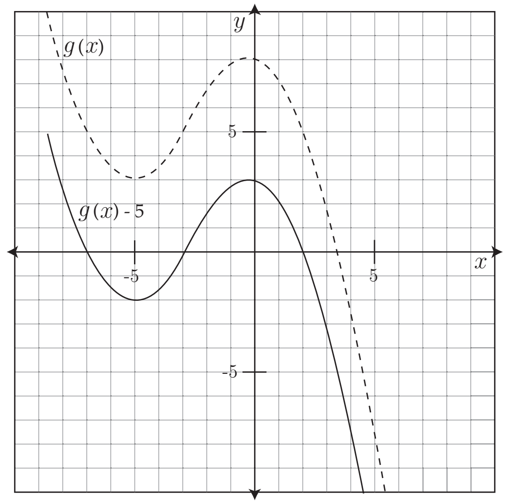
1. Find the x–intercepts of $f(x) = \cos(x^2 - 1)$ for $x \in \mathbb{R}$.

Solution: Since $\cos\theta = 0$ for $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots, \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$, it follows that $x^2 - 1 = \frac{\pi}{2} + n\pi \longrightarrow x^2 = 1 + \frac{\pi}{2} + n\pi$ so $x = \pm\sqrt{1 + \frac{\pi}{2} + n\pi}, n \in \mathbb{Z}_{\geq 0}$. □

2. Determine a reasonable equation for $g(x)$ below.

Solution: This is essentially finding the equation of a polynomial that crosses the x–axis, only we have to shift it down to do that.

Consider the graph labeled $g(x) - 5$ to see $g(x)$ shifted down 5 units. Now we can see the x–intercepts are $x = -7, -3$, and 2 so we have $g(x) - 5 = a(x + 7)(x + 3)(x - 2)$.
When $x = 0$, $g(x) - 5 = 3$ so the y–intercept is 3 and we solve to get $a = \frac{3}{-42} = \frac{-1}{14}$.
Then $g(x) - 5 = \frac{-1}{14}(x + 7)(x + 3)(x - 2)$. □



3. Complete the table below if f is linear, g is quadratic, and h is exponential.

t	-3	-2	-1	0	1	2	3
$f(t)$	7	3	-1	-5	-9	-13	-17
$g(t)$	7	3	-5	-17	-33	-53	-77
$h(t)$	7	3	$\frac{9}{7}$	$\frac{27}{49}$	$\frac{81}{343}$	$\frac{243}{2401}$	$\frac{729}{16807}$

4. A person's weight varies (approximately) with the cube of their height. If Larry is 5′8″ and weighs 160 pounds, how much would Aaron who is 6′2″ weigh assuming his build is of similar proportion.

Solution: $w = kh^3 \longrightarrow 160 = k(68)^3 \longrightarrow k \approx 0.0005089$.
Then $w \approx (0.0005089)h^3$
So $w \approx (0.0005089)(74)^3 = 206$ pounds. □

5. The base diameter of a tree (measured in cm) varies directly with the $\frac{3}{2}$ power of its height (measured in meters).

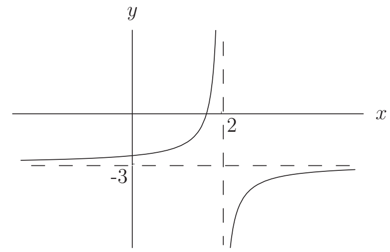
(a) If a tree 5 meters high has a base diameter of 14.5 cm, find the constant of proportionality and write a function relating the height and base diameter of any similar trees.

Solution: $d = kh^{3/2} \longrightarrow 14.5 = k(5)^{3/2} \longrightarrow k \approx 1.2969$.
Then $d \approx 1.2969h^{3/2}$. □

(b) Find the height of a tree with a base diameter of 238 cm.

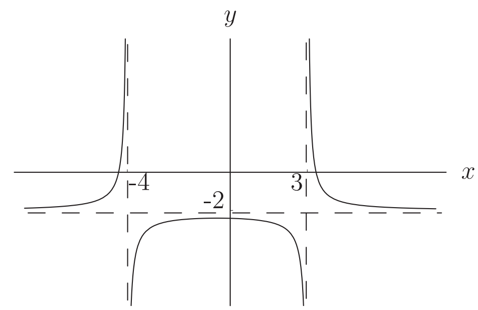
Solution: S0 $238 \approx 1.2969h^{3/2} \longrightarrow \frac{238}{1.2969} \approx h^{3/2} \longrightarrow h \approx (183.512)^{2/3} = 32.29$ meters. □

6. The graph below is a translation of the function $f(x) = \frac{1}{x}$. Find a possible formula for the graph and apply algebra to write it as the ratio of two linear functions.



Solution: Compared with the graph of $f(x) = \frac{1}{x}$, this looks like:
f flipped over the x–axis (so multiplied by -1);
shifted horizontally 2 spaces to the right $(x-2)$;
and shifted vertically down three spaces $(f(x)-3)$.
Therefore we have the new function: $y = \frac{-1}{x-2} - 3$.
Simplifying gives us $y = \frac{5-3x}{x-2}$. $\square$

7. Give a possible equation for the function graphed below.



Solution: This graph has vertical asymptotes at $x = -4$ and $x = 3$, usually a sign that we are dividing by zero at these points. Therefore we put $(x+4)(x-3)$ in the denominator.
Checking on the calculator shows that the graph of $y = \frac{1}{(x+4)(x-3)}$ looks similar but has its horizontal asymptote at $y = 0$, rather than at $y = -2$. The simple solution, then, is to shift $y = \frac{1}{(x+4)(x-3)}$ down two spaces by writing: $y = \frac{1}{(x+4)(x-3)} - 2$ or equivalently, $y = \frac{1-2(x+4)(x-3)}{(x+4)(x-3)} = \frac{-2x^2-2x+25}{x^2+x-12}$. $\square$

8. Determine the zeros, y-intercept, and asymptotes (vertical and horizontal) for the function below. Then sketch it.

$$g(x) = \frac{x^2-25}{x^2-4x-12}$$

Solution: Zeros occur where the numerator is 0 (and the denominator is not). So at $x^2 - 25 = 0 \longrightarrow x = \pm 5$.
The y–intercept occurs where $x = 0$ so at $\frac{-25}{-12} = 2\frac{1}{12}$.
$g(x)$ has vertical asymptotes where the denominator (and not the numerator) is 0 so where $x^2 - 4x - 12 = (x-6)(x+2) = 0$ or at $x = 6$ and $x = 2$. Horizontal asymptotes occur over the long term, so as x grows very large. Notice that as x gets big, the largest powers in the polynomials dominate so $x \to \infty : \frac{x^2-25}{x^2-4x-12} \to \frac{x^2}{x^2} = 1$. Therefore there is a horizontal asymptote at $y = 1$. $\square$

9. Suppose $f(1) = 5$ and $f(5) = 16$. Find a possible formula for f if f is

(a) linear
Solution: We have points (1, 5) and (5, 16) so the slope is
$m = \frac{16-5}{5-1} = \frac{11}{4}$.
Using the point (1, 5) we find b from our equation: $y = \frac{11}{4}x + b$:
$5 = \frac{11}{4}(1) + b \longrightarrow b = \frac{9}{4}$
So $f(x) = \frac{11}{4}x + \frac{9}{4}$. $\square$

(b) exponential
Solution: We have points (1, 5) and (5, 16) so inserting the values into the equation $y = a \cdot b^x$ gives us:
$16 = a \cdot b^5$ and $5 = a \cdot b^1$. Dividing the first equation by the second produces:
$\frac{16}{5} = b^4 \longrightarrow b = \sqrt[4]{\frac{16}{5}} = \left(\frac{16}{5}\right)^{1/4}$
Then we have $y = a \cdot \left(\left(\frac{16}{5}\right)^{1/4}\right)^x$ or equivalently $y = a \cdot \left(\frac{16}{5}\right)^{x/4}$, so
$5 = a \cdot \left(\frac{16}{5}\right)^{1/4} \longrightarrow a = 5\left(\frac{5}{16}\right)^{1/4} = \frac{5\sqrt[4]{5}}{2}$
So $f(x) = \frac{5\sqrt[4]{5}}{2} \cdot \left(\frac{16}{5}\right)^{x/4} \approx 3.74(1.34)^x$ $\square$

(c) power function
Solution: We have points (1, 5) and (5, 16) so inserting the values into the equation $y = k \cdot x^n$ gives us:
$16 = k \cdot 5^n$ and $5 = k \cdot 1^n$.
From the second equation we have $k = 5$ and substituting into the first gives us: $16 = 5 \cdot 5^n$.
Then $\frac{16}{5} = 5^n \longrightarrow \log\left(\frac{16}{5}\right) = n \log 5$
So $n = \frac{\log\left(\frac{16}{5}\right)}{\log 5} \approx 0.723$ and we have $f(x) \approx 5 \cdot x^{0.723}$. $\square$

10. Find $f^{-1}(x)$ for $f(x) = \frac{2x-1}{5-3x}$.

Solution: $y = \frac{2x-1}{5-3x} \longrightarrow x = \frac{2y-1}{5-3y} \longrightarrow x(5-3y) = 2y - 1 \longrightarrow 5x - 3xy = 2y - 1$
Then $5x + 1 = 2y + 3xy \longrightarrow 5x + 1 = y(2+3x) \longrightarrow y = \frac{5x+1}{2+3x}$ So $f^{-1}(x) = \frac{5x+1}{2+3x}$. $\square$