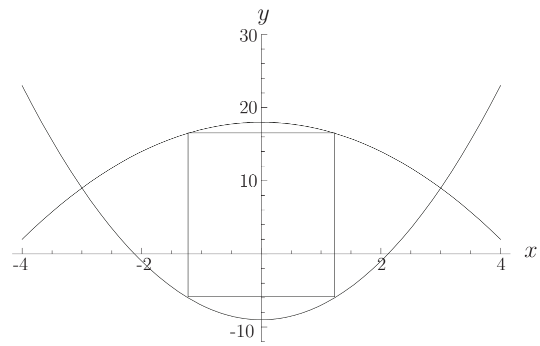
Optimization Problems

Show all relevant work!

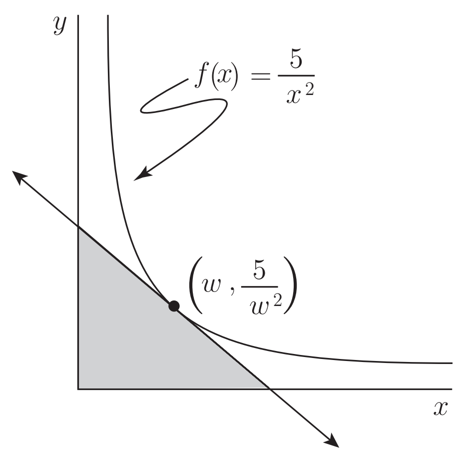
1. Consider the functions $f(x) = 18 - x^2$ and $g(x) = 2x^2 - 9$ shown below (see Figure 1). What is the area of the largest rectangle that can be inscribed inbetween these functions?



Answer: $36\sqrt{3}$

2. Consider the line tangent to the function $f(x) = \frac{5}{x^2}$ at the point $(w, \frac{5}{w^2})$.

Let $A(w)$ be the area of the triangle formed by the line tangent to $f(x)$ at $(w, \frac{5}{w^2})$, the x–axis, and the y–axis. Find the value of w that maximizes $A(w)$ on $[\frac{1}{2}, 3]$. Use calculus to explain this result.



Answer: $t = \frac{-10}{w^3}x + \frac{15}{w^2}$
$A(w) = \frac{45}{4w} \longrightarrow A'(w) = \frac{-45}{4w^2}$ so c.p. out of domain.
$A\left(\frac{1}{2}\right) = \frac{45}{2}$
$A(3) = \frac{15}{4}$ So max at $w = \frac{1}{2}$

3. Suppose you are given 100 meters of fence which you must use to form two separate corrals; one in the shape of a square, the other in the shape of a circle.

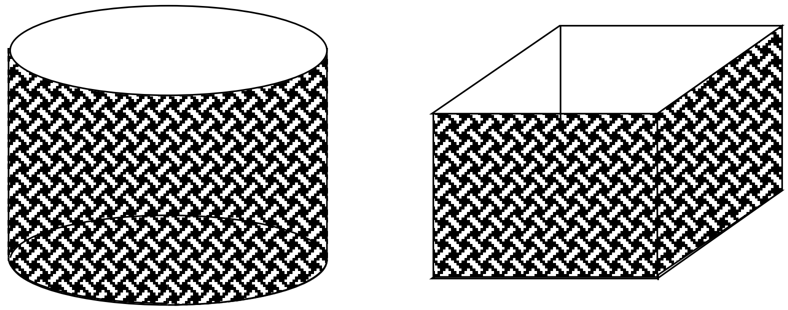


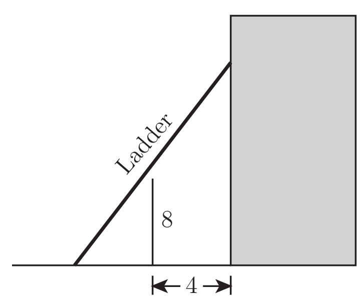
Figure 1: Circular and Square corrals

(a) What lengths must you cut in order to enclose the maximum total area?

(b) What is the minimum area that can be enclosed and under what circumstances?

Answer: $A'(x) = \frac{2x(\pi+4)-200}{\pi}$
c.p. at $x = \frac{100}{\pi+4}$
$A\left(\frac{100}{\pi+4}\right) = 350.06\text{m}^2$
$A(0) = 795.77$, $A(25) = 625$

4. A building is surrounded by an 8 foot fence placed 4 feet from the building. What is the shortest ladder you need in order to lean it against the building, over the fence?



Answer: Let x be the length of the base, then for $x = 10.35$, $h = 13.04$ we have $\ell = 16.65$ ft.