Problems

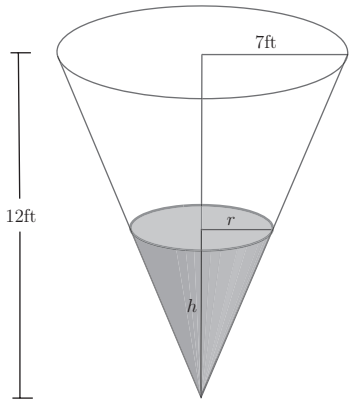
Show all relevant work!

You may use a calculator to verify solutions, but not to provide them.

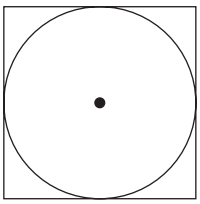
1. If each edge of a cube is increasing at the constant rate of 3cm/sec, how fast is the volume increasing when the length of an edge is 10cm long?

2. A public swimming pool has a rectangular shape with the following dimensions: length 30m, width 15m, depth 3m at the adult end and 1m at the children's end. If water is pumped in at 15 m^3/min, how fast is the water level rising when it is 2m deep at the adult end?

3. Water is draining out of a conical tank (see figure) in such a way that the height of the water is decreasing by 3 inches (= 0.25 ft) per minute. Find a general formula for the rate at which water is draining out of the cone (in cubic feet per minute). Use your result to help you find the rate at which the water is draining from the cone when the height of the water is 4 feet.



4. A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.



(a) Find the rate at which the *perimeter* of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

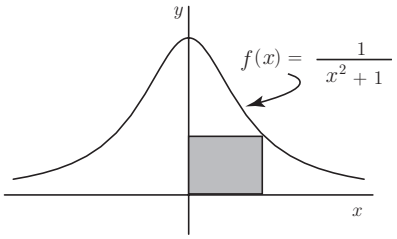
5. Air is being pumped into a spherical balloon at a rate of 10cc per second. How quickly is the surface area of the balloon increasing when the radius of the balloon is 6cm?

6. A builder wishes to fence in 60,000 m^2 of land in a rectangular shape. Because of security reasons, the fence along the front part of the land will cost \$2 per meter, while the fence along the other three sides will cost \$1 per meter. How much of each type of fence will the builder have to buy in order to minimize the cost of the fence? What is the minimum cost?

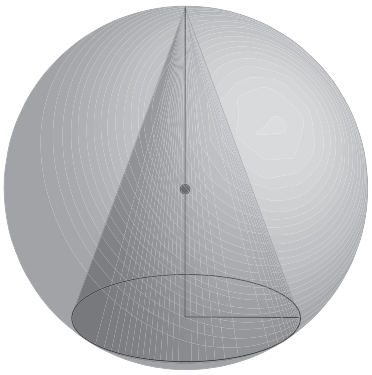
7. A cylindrical container with capacity 355cc is to be produced. The bottom and side of the container are to be made of material that costs 0.02 cents per cm^2, while the top of the container is made of material costing 0.03 cents per cm^2. Find the dimensions that will minimize the cost of the container. Provide a convincing argument that your results give the least expensive cost.

8. A rectangle is inscribed beneath the function $f(x) = \frac{1}{x^2+1}$ so that it is bounded by the x and y axes (see below).
(a)Determine the area of the largest rectangle that can be inscribed this way.

(b) Does the x value of this rectangle occur at an inflection point of $f(x)$? (Explain)

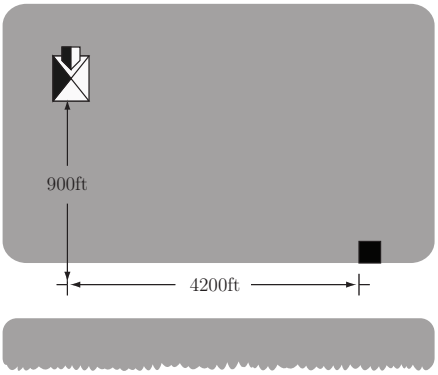


9. Determine the radius and height of the cone of greatest volume that can inscribed in a sphere of radius 10cm.



10. Show that for a fixed perimeter, k, the rectangle enclosing the largest area is a square.

11. A telephone company is asked to provide service to a customer whose house is located in the forest, 900 feet away from the road along which the phone lines run. The nearest telephone box is located 4,200 feet down the road. The cost to install the phone line is \$20 per foot along the road and \$35 per foot away from the road (through he forest).

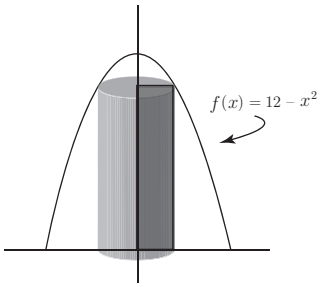


(a) How far from the box (along the road) should the company connect the telephone line so as to minimize installation costs?

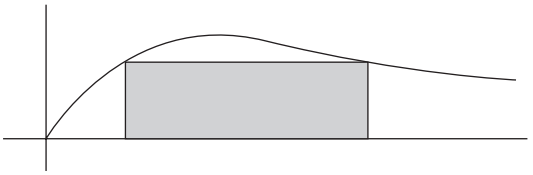
(b) What is the *least* it will cost to install the phone line?

(c) What is the *most* it will cost to install the phone line and under what conditions?

12. A cylinder is formed by rotating a rectangle about the y-axis. The base of the rectangle lies on the x-axis and the opposite corners are bounded by the function $y = 12 - x^2$. Find the maximum volume of the cylinder.



13. A cylinder is formed by rotating a rectangle about the x-axis. The base of the rectangle lies on the x-axis and the opposite corners are bounded by $f(x) = \frac{x}{x^2+1}$. Find the maximum volume of the cylinder.



14. A 20 foot piece of wire is to be cut into two pieces. One piece is used to form a circle and the other is used to form a square (imagine fence used to make a square corral and a circular corral). What lengths should be cut to make each shape in order to

(a) Maximize the total area enclosed.

(b) Minimize the total area enclosed.

15. A car rental agency has 24 cars (identical model). The owner of the agency finds that at a price of \$25 a day, all the cars can be rented; however, for each \$2 increase in rental, one of the cars is not rented. What should the agency charge to maximize income?