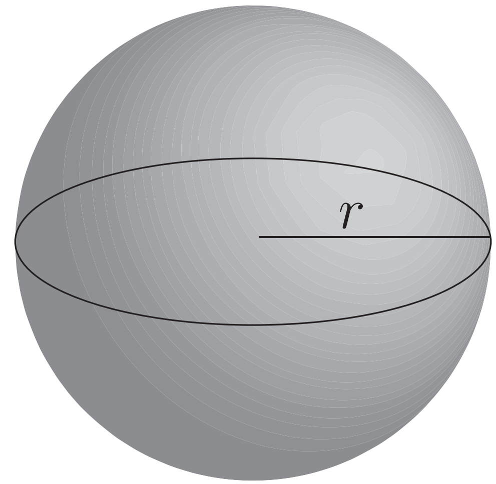
4.6 Notes

Related Rates Problems

The inflating sphere.

Air is being pumped into a spherical balloon at a rate of 30cc/sec. How quickly is the radius growing when the radius is 10cm?



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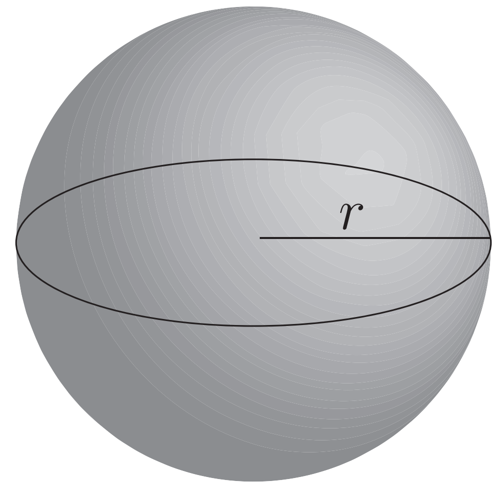

Solution:

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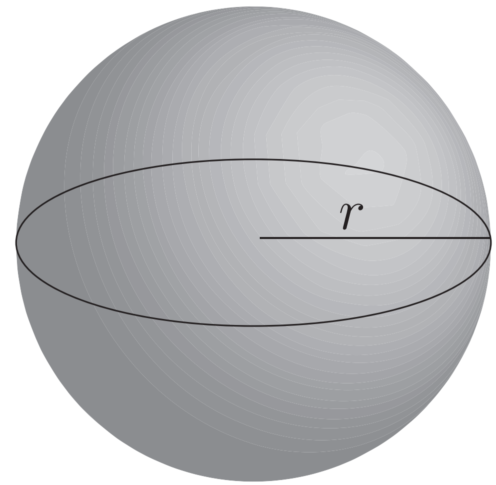

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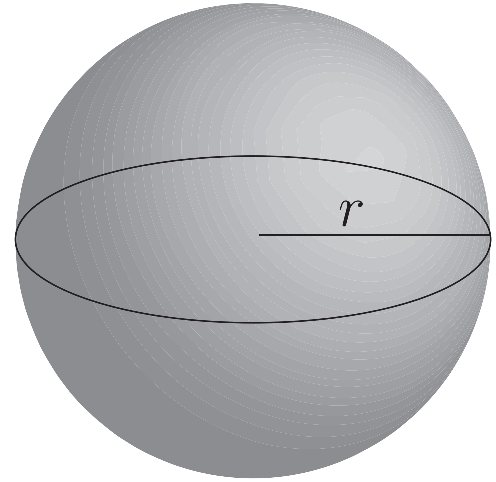

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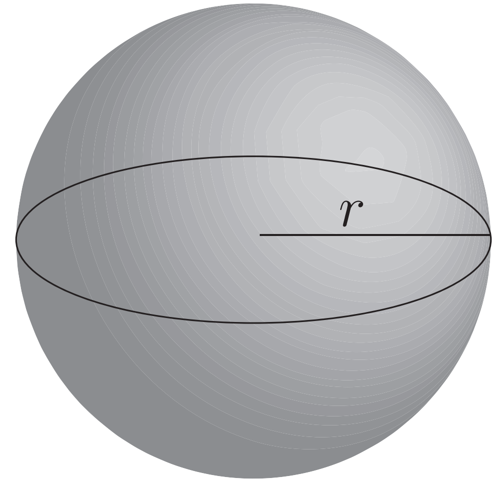

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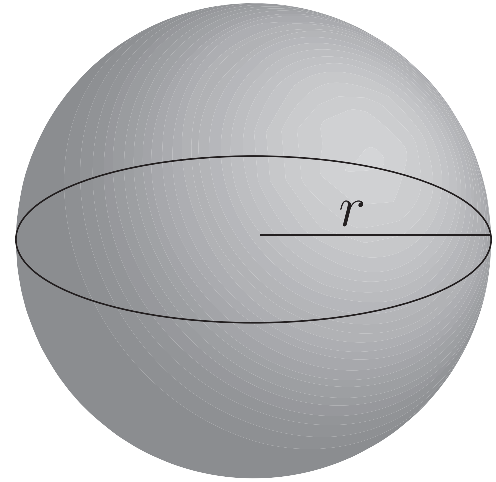

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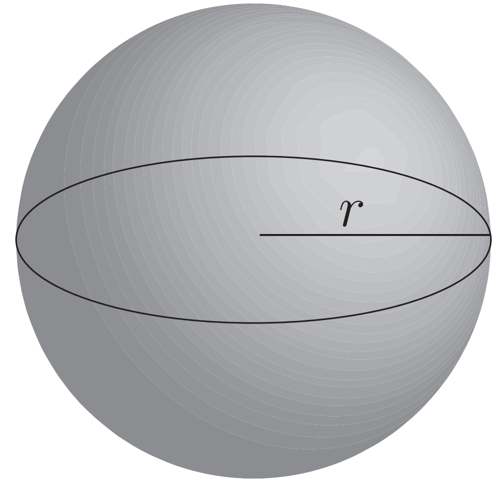

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Solving:

$$30 = 4\pi(10)^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{30}{400\pi} = \frac{\mathrm{d}r}{\mathrm{d}t}$$

So $\frac{\mathrm{d}r}{\mathrm{d}t} = 30/(400\pi) \approx 0.024$ cm/sec

The draining cone.

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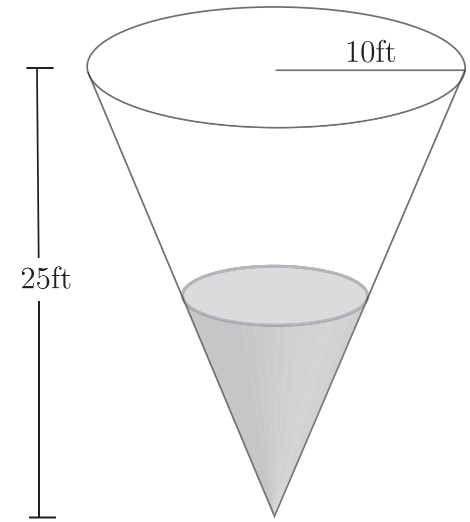

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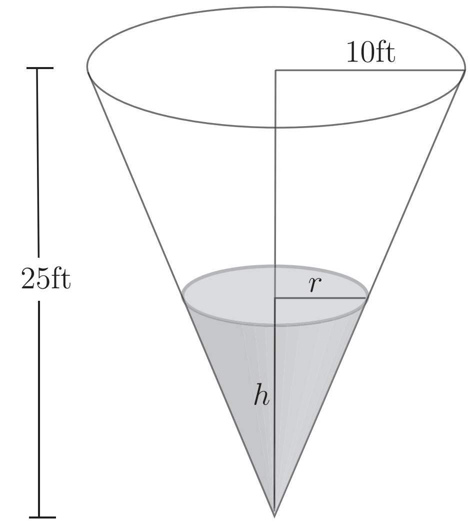

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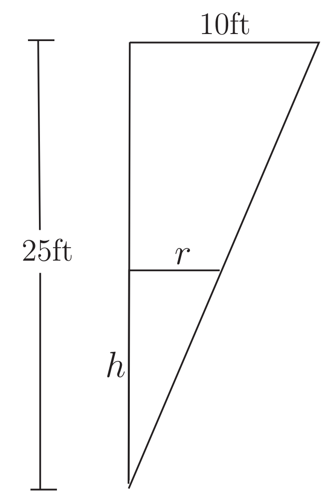

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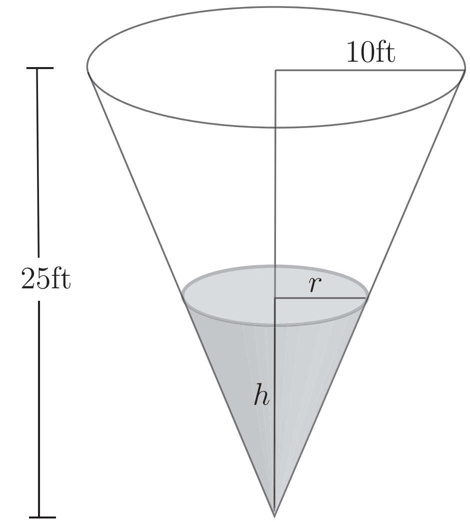

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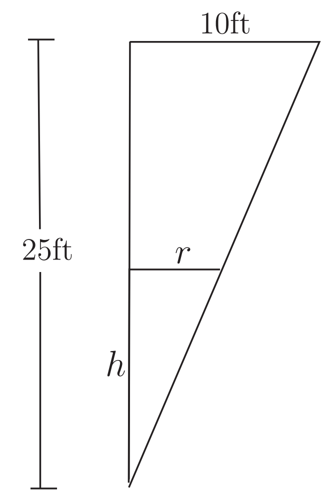

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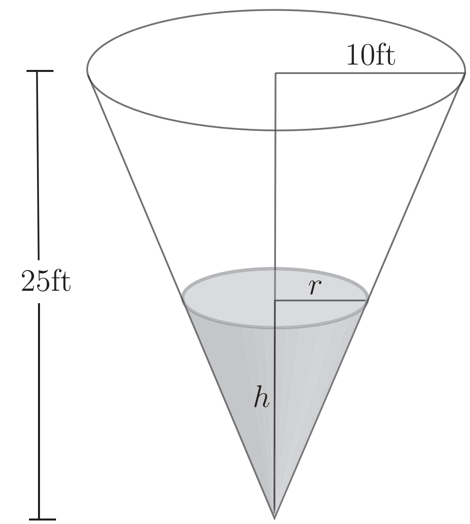

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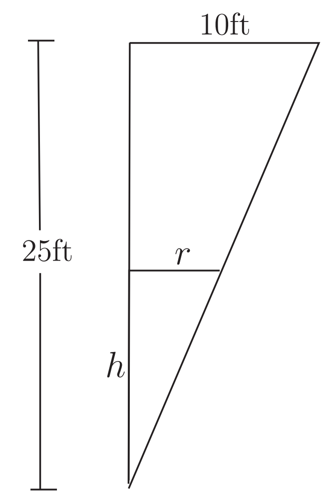

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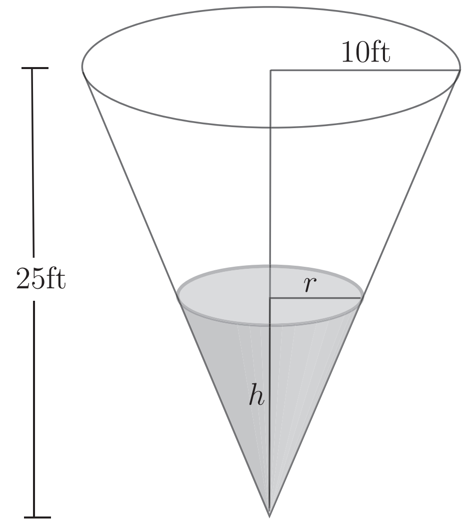

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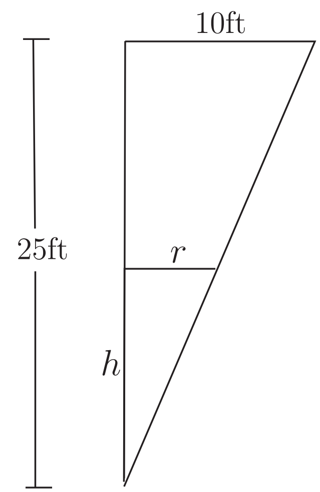

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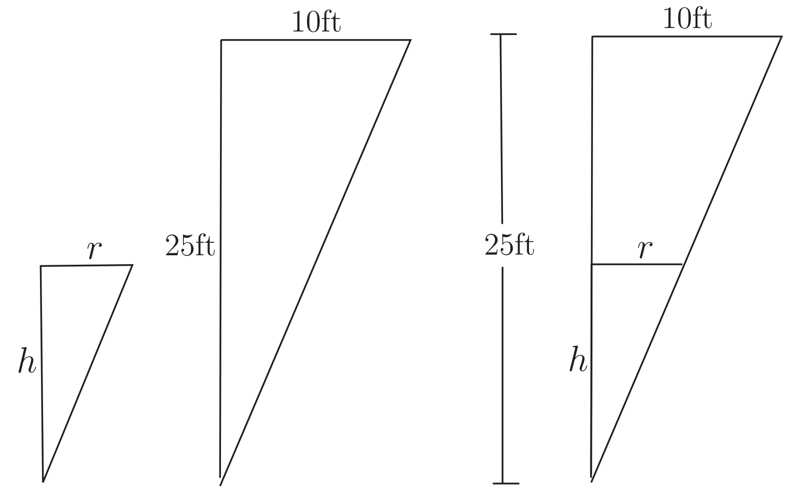

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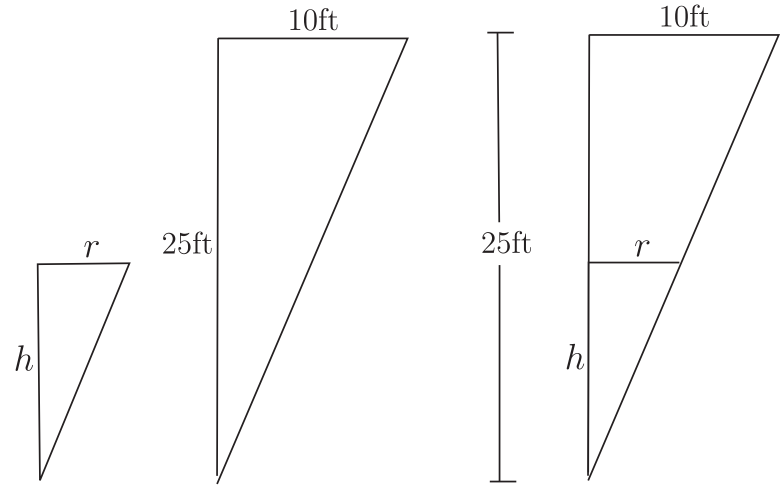

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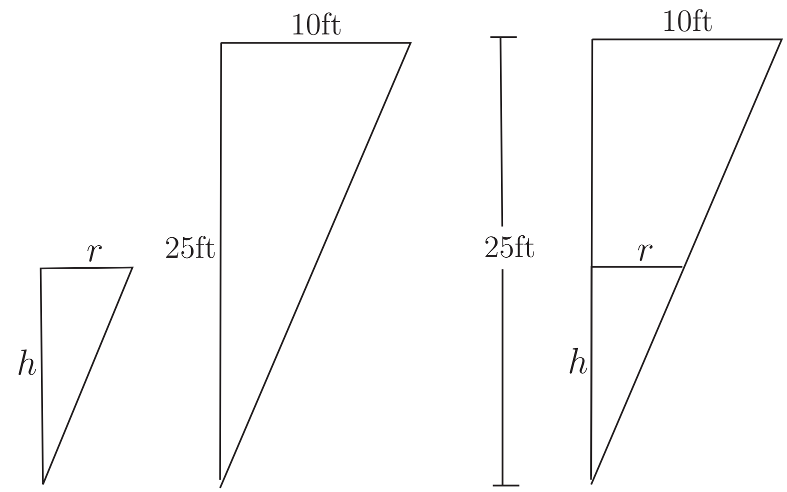

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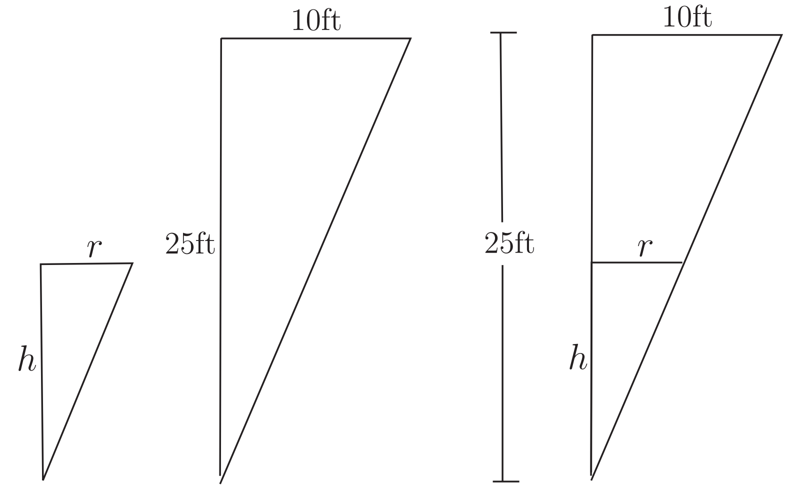

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So $-0.5 = \dfrac{4\pi}{25}(8)^2 \cdot \dfrac{\mathrm{d}h}{\mathrm{d}t} \longrightarrow \dfrac{\mathrm{d}h}{\mathrm{d}t} = -\dfrac{25}{512\pi}$ ft/min.

Or dropping by a little over $\dfrac{3}{16}''$ per minute.

Related Rates Problems

The speed trap.

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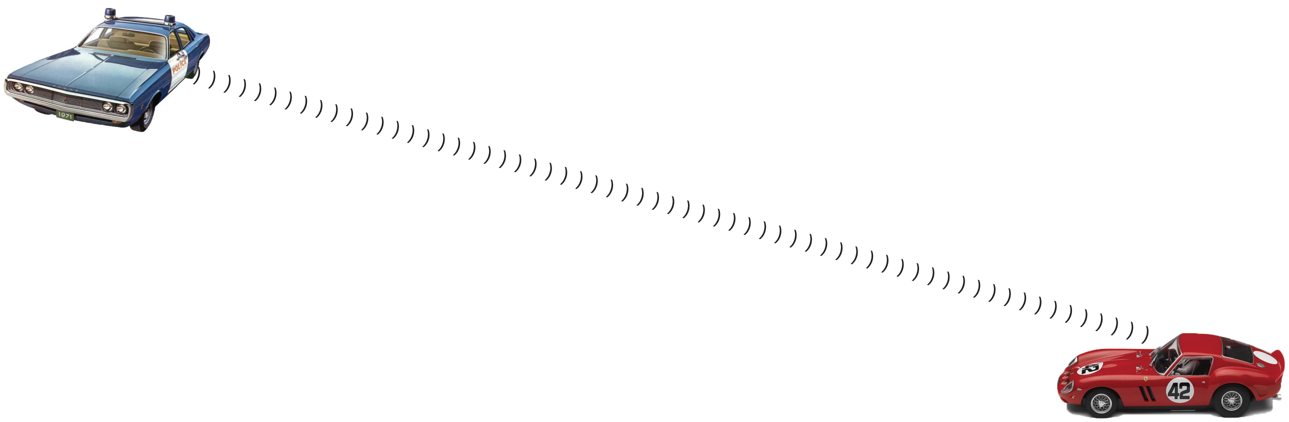

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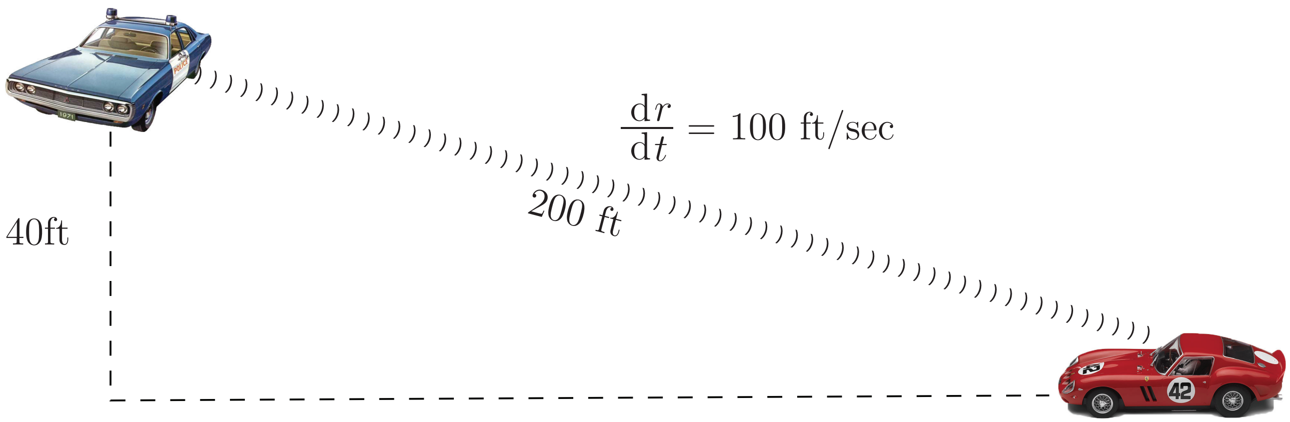

When he pulls you over he claims that you were actually traveling about 70 mph on the road.
Is this possible?!