

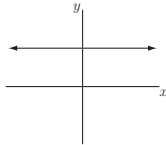
## 3.1 Notes

1. Some elementary derivatives.

$\frac{d}{dx}k$ : **The derivative of a constant.** Some arguments for  $\frac{d}{dx}k = 0$ :

Graphically:

The derivative gives the slope of the function at  $x$ .  
For a constant function, the slope is 0 so  $\frac{d}{dx}k = 0$ .



Algebraically (from definition):

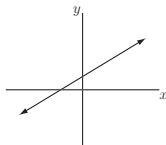
$$\frac{d}{dx}k = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{k - k}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$\frac{d}{dx}(mx + b)$ : **The derivative of a linear function.**

Some arguments for  $\frac{d}{dx}(mx + b) = m$ :

Graphically:

The derivative gives the slope of the function at  $x$ .  
For a linear function, the slope is  $m$  so  $\frac{d}{dx}(mx + b) = m$ .



Algebraically (from definition):

$$\begin{aligned} \frac{d}{dx}(mx + b) &= \lim_{h \rightarrow 0} \frac{m(x+h) + b - (mx + b)}{h} \\ &= \lim_{h \rightarrow 0} \frac{mh}{h} = \lim_{h \rightarrow 0} m = m \end{aligned}$$

2. Some derivative properties

$\frac{d}{dx}kf(x)$ : **The derivative of a constant times a function.**

Numerical intuition: Find the slopes between each pair of points to approximate the derivative of  $f(x)$ :

$x$	-3	0	3	6	9
$f(x)$	10	8	1	5	11

Now repeat for the function  $y = 5f(x)$ :

$x$	-3	0	3	6	9
$5f(x)$	50	40	5	25	55

How are the slopes of the two tables related?

The proof follows similar logic but uses limit properties and the definition of a derivative function which may make it seem more abstract.

Remember the definition of the derivative function is  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

**Proof:**

$$\begin{aligned} \frac{d}{dx}kf(x) &= \lim_{h \rightarrow 0} \frac{kf(x+h) - kf(x)}{h} \\ &= \lim_{h \rightarrow 0} k \frac{f(x+h) - f(x)}{h} \\ &= k \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= kf'(x) \end{aligned}$$

□

$\frac{d}{dx}(f(x) + g(x))$ : **The derivative of the sum of functions.**

Numerical intuition: Find the slopes between each pair of points to approximate the derivatives of  $f(x)$  and  $g(x)$ :

$x$	-3	0	3	6	9
$f(x)$	10	8	1	5	11

$x$	-3	0	3	6	9
$g(x)$	-2	1	6	15	27

Now repeat for the function  $y = f(x) + g(x)$ :

$x$	-3	0	3	6	9
$f(x) + g(x)$	8	9	7	20	38

How are the slopes of  $f$ ,  $g$ , and  $f + g$  related?

The proof follows:

**Proof:**

$$\begin{aligned} \frac{d}{dx}(f(x) + g(x)) &= \lim_{h \rightarrow 0} \frac{(f(x+h) + g(x+h)) - (f(x) + g(x))}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ &= f'(x) + g'(x) \end{aligned}$$

□

The same argument applies to  $\frac{d}{dx}(f(x) - g(x))$  so in general we say  $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$ .

3. You might wonder if the same is true of the product of two functions,  $\frac{d}{dx}(f(x) \cdot g(x))$ . As before, it's worth some time to explore intuitively first so reuse the tables above to see if  $\frac{d}{dx}(f(x) \cdot g(x)) \stackrel{?}{=} f'(x) \cdot g'(x)$ .

$x$	-3	0	3	6	9
$f(x)$	10	8	1	5	11

$x$	-3	0	3	6	9
$g(x)$	-2	1	6	15	27

$x$	-3	0	3	6	9
$f(x) \cdot g(x)$	-20	8	6	75	297

What does this suggest is (or isn't) true about  $\frac{d}{dx}(f(x) \cdot g(x))$ ?

4. **Power Rule Examples:**

(a)  $\frac{d}{dx}(5x^2)$

(b)  $\frac{d}{dx}\left(\pi x^5 - \frac{2}{x^3}\right)$

(c)  $\frac{d}{dx}\left(\frac{1}{x} - \frac{3}{\sqrt{x}} + \sqrt[5]{x^3}\right)$

(d)  $\frac{d}{dx}\left(\frac{x^4 - 7x}{x^2}\right)$