This is the original document. Please see last page for typeset of Dr. Math's "proof". Your job is to identify the error in the good doctor's argument. Review your limit properties first! Submit your response identifying where the error occurs and explain why this is a flaw in logic.

Derivative of e^x by Limit Definition

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Date: 05/21/2002 at 22:40:18
From: Jeff King
Subject: derivative of e^x by limit definition

Dear Dr. Math,

I've tried (as has my entire calculus class) to prove that the
derivative of e^x is e^x by the limit definition of the derivative
(without using the Taylor expansion for e^x) and we cannot seem to
get past one last step.

In setting up the form of the limit definition of the derivative we've
made e(x+delta x) into e^x*e^(delta x) for convenience and left the
-e^x in the numerator of the limit definition. Then by factoring we
determined that the limit as delta x goes to 0 is

  (e^x(e^(delta x)-1))/delta x.

The e^x alone in the numerator would seem to be our best bet, but we
can't make the rest of the function go to 1 as some people have
guessed it should.

How would one prove the limit definition or just that

  (e^(delta x)-1)/(delta x) = 1

as x tends to infinity?  Thanks for your assistance!
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Date: 05/29/2002 at 22:41:49
From: Doctor Fwg
Subject: Re: derivative of e^x by limit definition

Dear Jeff,

Here is a possible solution that does not involve the use of a Taylor
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expansion.  I hope the notation I have used here will be easy enough
to follow.

Definition of e is:

  1)  e = Limit   (1 + 1/n)^n
           n->Inf

However, if n = 1/h, then

  2)  e = Limit   (1 + h)^(1/h)
           h->0

If f(x) = e^x, the definition of f'(x) is:

  3)  f'(x) = Limit  [f(x + h) - f(x)]/h
               h->0

But:

  4)  f(x + h) = e^(x+h) = (e^x)(e^h)

So:

  5)  f'(x) = Limit [(e^x)(e^h) - e^x]/h
               h->0

            = Limit [(e^x)((e^h) - 1)]/h
               h->0

But, raising both sides of equation (2) to the power of h yields:

  6)  e^h = Limit [(1 + h)^(1/h)]^h
             h->0

          = Limit [(1+ h)]
             h->0

Placing this value of e^h into equation (5) yields:

  7)  f'(x) = Limit [(e^x){(1 + h) - 1}]/h
               h->0

            = Limit [e^x(h/h)]
               h->0

But, h/h = 1, so:

  8)  f'(x) = Limit [e^x(1)]
               h->0

            = e^x

With Best Regards,

Doctor Fwg, The Math Forum
http://mathforum.org/dr.math/
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Associated Topics:

Dr. Fwg's "proof"

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \tag{1}$$

$$n = \frac{1}{h} \longrightarrow e = \lim_{h\to 0}(1+h)^{\frac{1}{h}} \tag{2}$$

$$f'(x) = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h} \tag{3}$$

$$f(x+h) = e^{x+h} = e^x e^h \tag{4}$$

$$f'(x) = \lim_{h\to 0}\frac{e^x e^h - e^x}{h} \tag{5}$$

$$e^h = \left(\lim_{h\to 0}(1+h)^{\frac{1}{h}}\right)^h = \lim_{h\to 0}(1+h) \tag{6}$$

$$f'(x) = \lim_{h\to 0}\frac{e^x(1+h-1)}{h} = \lim_{h\to 0}\frac{e^x h}{h} \tag{7}$$

$$f'(x) = \lim_{h\to 0} e^x(1) = e^x \tag{8}$$