Functions: Domain and Range, Intro Linear functions

Domain:

The domain of a function is the number set to which the independent variable belongs. Let's review some number sets:

Natural ($\mathbb{N}$) $= \{1, 2, 3, \dots\}$
Integer ($\mathbb{Z}$) $= \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
Rational ($\mathbb{Q}$) = Set of numbers that can be expressed as $\frac{a}{b}$ where a and b are integers.
Real ($\mathbb{R}$) = Set of Rationals combined with **irrationals** (numbers with infinite, non-repeating decimals such as π or $\sqrt{2}$).
Complex ($\mathbb{C}$) = Set of numbers in form $a + bi$ where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$.

We typically neglect to specify a domain when defining a function, therefore assuming that the domain is $\mathbb{R}$. Note, however, if $B = f(n) = n/33$ is the number of school busses needed as a function of the number of students in a school district (technically $S = f(n) = \lceil n/33 \rceil$.), the domain for B is $\mathbb{N}$ and the range is also $\mathbb{N}$, written $f : \mathbb{N} \longrightarrow \mathbb{N}$.

1. Determine the best set(s) of numbers suited to the domain and range of these functions.

   (a) The amount of money an electrician makes as a function of time in hours.

   (b) The population of a town as a function of time in years.

   (c) The number of houses sold by a realtor as a function of the selling price.

   (d) The temperature as a function of the date in a year.

2. (a) Sketch graphs of (i) an increasing function and (ii) a decreasing function.

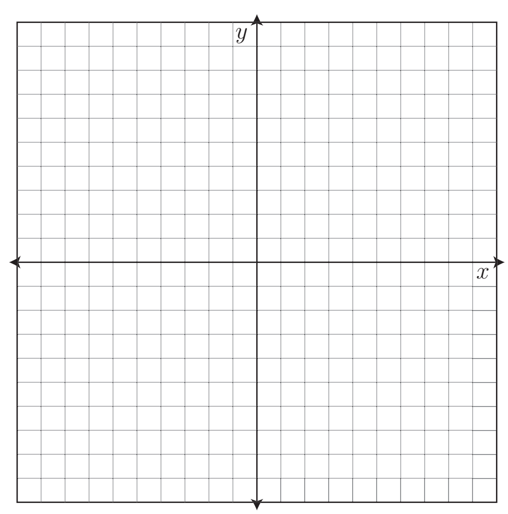
   (b) Write definitions for increasing and decreasing functions in terms of the independent variable (x) and the dependent variable (y).

Linear Functions:

3. Complete the table below:

x	-6	-3	0	3	6
$y = \frac{2}{3}x + 5$					

4. Sketch the graph of the table in (3).



5. Take a moment to note how the changes in the x and y variables in the two previous questions are manifested.

6. Use the table below to answer the following questions:

x	0	1	2	3	4
$f(x)$	9	8	5	0	-7

(a) find $f(4)$: ___________

(b) Solve $f(x) = 0$: $x =$___________

(c) Find the average rate of change from $x = 0$ to $x = 2$ and then from $x = 2$ to $x = 4$. What do you observe from your results?

7. The population of Half Moon Bay since 1995 can be modeled by the function $P(t) = 10000(1.012)^t$ where t is in years. Find the average rate of growth in population from 1995 to 1997 and from 1999 to 2001. What do you observe from your results?

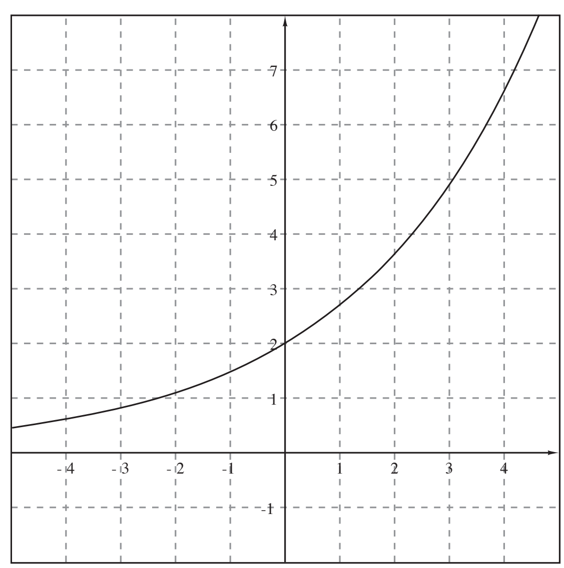
8. Use the exponential function, $g(x)$, graphed below to answer the given questions.

(a) Estimate the average rate of change between $x = -2$ and $x = 0$.

(b) Estimate the average rate of change between $x = 0$ and $x = 2$.

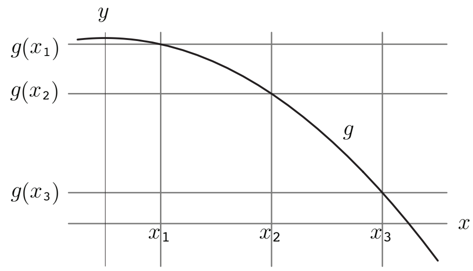
(c) Estimate $g(1)$.

(d) Estimate the solution to $g(x) = 7$.

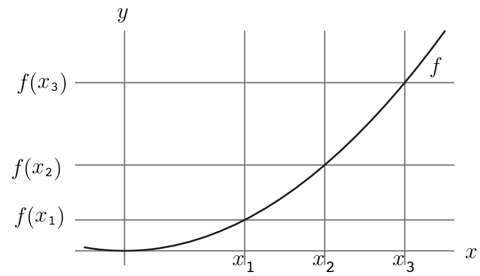


9. Write formulas for the average rate of change from x_1 to x_2 and from x_2 to x_3 in each function. Discuss the qualitative changes in rates of change (positive to negative, relative size).

(a)



(b)



10. The number of firearm-related deaths in the United States for various years are shown in the table below.

Firearm-Related U.S. Deaths

Year	Firearm-Related Deaths (thousands of deaths)
1993	40
1995	36
1997	32
1999	29
2000	28

Source: *Centers for Disease Control & Prevention*

(a) Use your graphing calculator to plot a scattergram of the data. Then use the space below to find the equation of a line that approximates the data well.
Let n represent the number (in thousands) of firearm-related deaths t years since 1990.

(b) What is the slope of your model and what does it mean in this situation?

(c) What is the n-intercept and what does it mean in this situation?

(d) When does your model predict there will be no firearm -related deaths in the U.S.?

5. Find the formula for the function table below.

x	-7	-2	3	8	13
y	13	10	7	4	1

6. A taxi charges $2 for every five miles traveled and a $3 pickup charge.
Write the cost of a cab ride as a function of the distance traveled. How far can you travel for $20?

A constant rate of change characterizes linear functions. Consider secton 1.2 #15 for example - is this linear? How do we know from the points?
What about the table below, is this a linear function? Notice that it's the ratio of terms rather than than just looking at how x and y grow separately.

x	-8	-2	0	4	10
y	-31	-10	-3	11	32

This tells us something about the behavior of linear functions in real situations: relationships where both quantities increase (decrease) at a constant rate. For instance, $ per hour,$ per pound, miles per hour, $ per mile. e.g. you make $9/hr means that for every hour you add to your time working, you earn 9 more $. etc.
Ex: The population of Largetown as a function of time in years since 1950 is given by $P(t) = 300t + 2000$. Explain the meanings of both parameters.

1.4
The last examples model what we call a direct proportion. Specifically, a situation where one quantity increases (decr.) at a constant rate as another quantity increases (decr.) at a constant rate. So a direct proportion is a special case of a linear function (where the initial value is 0). A direct proportion is written $y \propto x$ and it means almost equal but maybe as one increases by 1 the other might increase by a different amount - say k. To write an equation you need to say $y = kx$. Similarly, for a proportion where y varies with the cube of x we say $y \propto x^3$ so $y = kx^3$.

Have them attempt and discuss:
a) What is the equation of the line through $(4, -5)$ and $(10, 4)$?

b) What is the equation of the line through $(6, 7)$ and perpendicular to $3x - 2y = 17$

c) The strength, s, of a beam is proportional to the square of its thickness, h.

d) Suppose your water bill works like this. For 1000 cubic feet you pay $90 but for 1600 cu.ft. you pay $105.

(i) What is the charge per cubic foot? (ii) Write an equation for cost as a function of water quantity.

Follow with: Write the equation of the line perpendicular to $3x - 7y = 19$ and containing $(12, -5)$.

On side function types and graphs: linear: , power w/ odd/even exponents , exponential .
Context: In general this tells us something about the behavior of linear functions in real situations: relationships where the rate of change comparing two quantities increases (decreases) at a constant rate. For instance, $ per hour, $ per pound, miles per hour,$ per mile. e.g. you make $9/hr means that for every hour you add to your time working, you earn 9 more $. etc.

Ex.1 You ride your bicycle out of town. After two hours you've gone 30 miles and after 5 hours you've gone 77 miles. Write a formula giving distance as a function of time.

Discuss the significance of the d–intercept.

Ex.2 Suppose your water bill works like this. For 1000 cubic feet you pay $90 but for 1600 cu.ft. you pay $105.

(i) What is the charge per cubic foot?

(ii) Write an equation for cost (C) as a function of water quantity (q). WHAT are your assumptions?

(iii) Find $C(1200)$ and $C(1201)$