Trigonometry Notes

# 0.1 Graphing

The standard training in every trig class teaches us that the table for $y = \sin x$ looks something like:

| $x$ | $-2\pi$ | $-\frac{3\pi}{2}$ | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $y = \sin x$ | 0 | 1 | 0 | $-1$ | 0 | 1 | 0 | $-1$ | 0 |

This emphasizes the maximum value ($y = 1$) and the minimum value ($y = -1$) as well as the *center line* (or midline) between them ($y = 0$). From this we take the definitions of *amplitude* which is just the distance between the center line and the maximum or minimum as well as the *period* which is the distance (or time) before the function's outputs begin repeating. The related concept of *frequency* is the number of cycles completed per unit time and follows from the reciprocal of the period.

We introduce the usual manipulations that shift, reflect, and stretch or compress the function: $f(x) = A\sin(Bx + C) + D$ and turn our attention to the question of identifying all of the essential points under these effects.

## 0.1.1 Vertical Effects

Vertical translations, reflections, and stretches are all based on changes to the output of a function so our focus is on changing the output values of a sine or cosine function.

- **Shifts**

  The table form for a function like $g(x) = 4 + \sin x$ looks like:

| $x$ | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|
| $\sin x$ | 0 | $-1$ | 0 | 1 | 0 |
| $y = 4 + \sin x$ | 4 | 3 | 4 | 5 | 4 |

  Note the intermediate step where we find the sine value first.

- **Reflections and stretches**

  Like vertical shifts, a vertical reflection or stretch affects the output values but this time through multiplication rather than addition. The table form for a function like $h(x) = -\frac{1}{2}\cos x$ looks like:

| $x$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|
| $\cos x$ | 1 | 0 | $-1$ | 0 | 1 |
| $y = -\frac{1}{2}\cos x$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ |

## 0.1.2 Horizontal Effects

Horizontal translations, reflections, and stretches are all based on changes to the input of a function so our focus is on working backwards from the input values of a sine or cosine function.

As a rule, begin with the standard table for sine or cosine and work backwards to find the corresponding input value:

| $x$ | | | | | |
|---|---|---|---|---|---|
| ( ) | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ |
| sin( ) | 0 | $-1$ | 0 | 1 | 0 |

- **Shifts**

  For example, the table for $f(x) = \sin(x - 2)$ is started below:

| $x$ | | | | | |
|---|---|---|---|---|---|
| ( ) | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ |
| sin( ) | 0 | $-1$ | 0 | 1 | 0 |

  Since $x - 2$ fills the parentheses, we solve $x - 2 = -\pi$, $x - 2 = -\frac{\pi}{2}$ and so on to find the input values for $x$.

- **Reflections and stretches**

  Like horizontal shifts, a horizontal reflection or stretch affects the input values but this time through multiplication rather than addition. For example, a function like $k(x) = \cos\left(-\frac{1}{2}x\right)$ begins with:

| $x$ | | | | | |
|---|---|---|---|---|---|
| ( ) | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| $y = \cos\left(-\frac{1}{2}x\right)$ | $1$ | $0$ | $-1$ | $0$ | $1$ |

  Solving $-\frac{1}{2}x$ for each of the values in the parentheses gives the $x$ input values:

$$-\tfrac{1}{2}x = 0 \to x = 0 \qquad -\tfrac{1}{2}x = \tfrac{\pi}{2} \to x = -\pi \qquad \ldots$$

| $x$ | $0$ | $-\pi$ | $-2\pi$ | $-3\pi$ | $-4\pi$ |
|---|---|---|---|---|---|
| ( ) | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| $y = \cos\left(-\frac{1}{2}x\right)$ | $1$ | $0$ | $-1$ | $0$ | $1$ |

- **Combinations**

  For functions involving both input and output manipulation, start from the heart of the function and work outwards. e.g. $f(x) = 5 - 3\sin(2x - \frac{\pi}{4})$ starts with the table like above:

| $x$ | | | | | |
|---|---|---|---|---|---|
| ( ) | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ |
| sin( ) | $0$ | $-1$ | $0$ | $1$ | $0$ |
| $y = 5 - 3\sin(2x - \frac{\pi}{4})$ | | | | | |

  Working backwards as above, we find the $x$ inputs corresponding to the ( ) values:

| $x$ | $-\frac{3\pi}{8}$ | $-\frac{\pi}{8}$ | $\frac{\pi}{8}$ | $\frac{3\pi}{8}$ | $\frac{5\pi}{8}$ |
|---|---|---|---|---|---|
| ( ) | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ |
| sin( ) | $0$ | $-1$ | $0$ | $1$ | $0$ |
| $y = 5 - 3\sin(2x - \frac{\pi}{4})$ | | | | | |

  Finally, using the values from sin( ) we finish the arithmetic to get:

$$2x - \tfrac{\pi}{4} = -\pi \to x = -\tfrac{3\pi}{8} \qquad 2x - \tfrac{\pi}{4} = -\tfrac{\pi}{2} \to x = -\tfrac{\pi}{8} \qquad \ldots$$

| $x$ | $-\frac{3\pi}{8}$ | $-\frac{\pi}{8}$ | $\frac{\pi}{8}$ | $\frac{3\pi}{8}$ | $\frac{5\pi}{8}$ |
|---|---|---|---|---|---|
| ( ) | $-\pi$ | $\frac{-\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ |
| sin( ) | $0$ | $-1$ | $0$ | $1$ | $0$ |
| $y = 5 - 3\sin(2x - \frac{\pi}{4})$ | $5$ | $8$ | $5$ | $2$ | $5$ |

  The top and bottom values provide the ordered pairs of the function. Notice that the period is relatively easy to identify from the top values (since these are derived from the period of $\sin x$) and we can give a second period of the function by following the pattern in these numbers.

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Show all relevant work!

## 0.2 Exercises

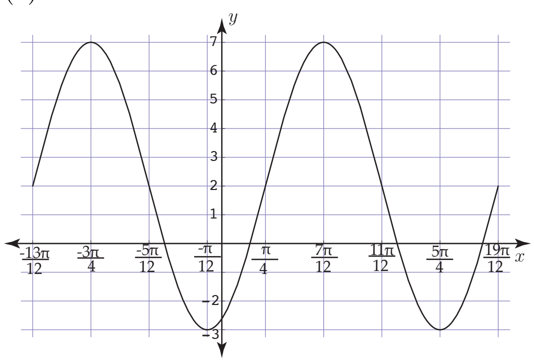
1. Sketch and label the following functions over two periods.

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = 2 + \sin x$ (d) $y = \sin(x+2)$

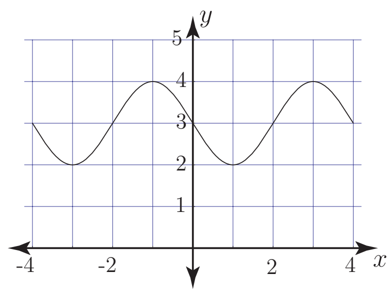
(e) $y = 2\sin x$ (f) $y = \sin(2x)$ (g) $y = \sin(2x-2)$ (h) $y = 3\cos\left(\frac{\pi}{4}x\right) - 1$

2. The graphs shown below match equations of the form $f(x) = A\sin(Bx+C)+D$ or $f(x) = A\cos(Bx+C)+D$ . Find a suitable equation for each graph.

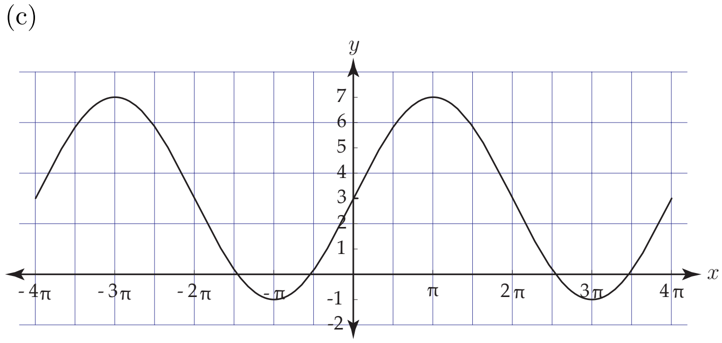
(a)




(b)




(c)




(d)