

40 CHAPTER 1 Linear Equations and Linear Functions

Find an approximate equation of the line that passes through the two given points. Round the slope and the constant term to two decimal places. Use a graphing calculator to verify your result.

- 37. $(-5.1, -3.9)$ and $(7.4, 2.2)$
- 38. $(-9.4, 7.1)$ and $(3.9, -2.3)$
- 39. $(-5.97, -6.24)$ and $(-1.25, -4.05)$
- 40. $(-7.13, -2.21)$ and $(-4.99, -7.78)$

Find an equation of the line that contains the given point and is parallel to the given line. Use a graphing calculator to verify your result.

- 41. $(4, 5)$, $y = 3x + 1$
- 42. $(1, 4)$, $y = 4x - 6$
- 43. $(-3, 8)$, $y = -2x + 7$
- 44. $(2, -3)$, $y = -x + 2$
- 45. $(4, 1)$, $y = \frac{1}{2}x - 3$
- 46. $(6, -3)$, $y = -\frac{2}{3}x - 1$
- 47. $(3, 4)$, $3x - 4y = 12$
- 48. $(4, -1)$, $5x + 2y = 10$
- 49. $(-3, -2)$, $6y - x = -7$
- 50. $(-1, -4)$, $3y + 5x = -11$
- 51. $(2, 3)$, $y = 6$
- 52. $(3, -1)$, $y = -4$
- 53. $(-5, 4)$, $x = 2$
- 54. $(-2, -5)$, $x = 1$

Find an equation of the line that contains the given point and is perpendicular to the given line. Use ZStandard followed by ZSquare with a graphing calculator to verify your result.

- 55. $(3, 8)$, $y = 2x + 5$
- 56. $(2, 1)$, $y = 5x - 4$
- 57. $(-1, 7)$, $y = -3x + 7$
- 58. $(-3, -2)$, $y = -6x - 13$
- 59. $(2, 7)$, $y = -\frac{2}{5}x + 3$
- 60. $(1, -2)$, $y = \frac{1}{3}x - 4$
- 61. $(10, 3)$, $4x - 5y = 7$
- 62. $(6, -1)$, $5x + 2y = -9$
- 63. $(-3, -1)$, $-2x + y = 5$
- 64. $(-1, 2)$, $-3x - 4y = 12$
- 65. $(2, 3)$, $x = 5$
- 66. $(-4, -2)$, $x = -1$
- 67. $(2, 8)$, $y = -3$
- 68. $(1, -1)$, $y = 7$

- 69. Let y be the value (in thousands of dollars) of a car when it is x years old. Some pairs of values of x and y are listed in Table 22.

| Age (years) | Value (thousands of dollars) |
|-------------|------------------------------|
| x | y |
| 0 | 19 |
| 1 | 17 |
| 2 | 15 |
| 3 | 13 |
| 4 | 11 |

Find an equation that describes the relationship between x and y .

- 70. Let y be a person's salary (in thousands of dollars) after he has worked at a company for x years. Some pairs of values of x and y are listed in Table 23.

| Time at Company (years) | Salary (thousands of dollars) |
|-------------------------|-------------------------------|
| x | y |
| 0 | 25 |
| 1 | 28 |
| 2 | 31 |
| 3 | 34 |
| 4 | 37 |
| 5 | 40 |

Find an equation that describes the relationship between x and y .

- 71. Find an equation of the line sketched in Fig. 96. Check your equation with a graphing calculator.

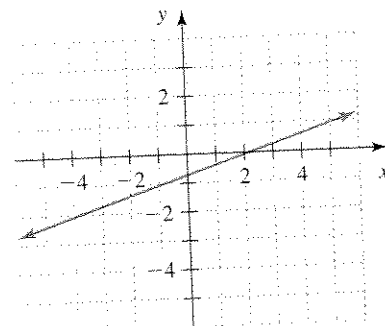


Figure 96 Exercise 71

- 72. Find an equation of the line sketched in Fig. 97. Check your equation with a graphing calculator.

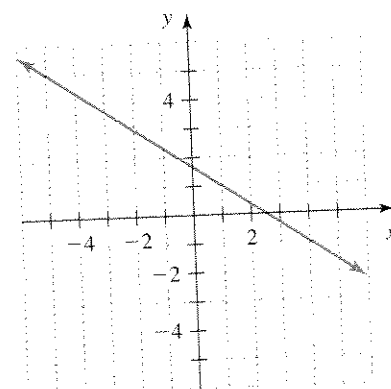


Figure 97 Exercise 72

- 73. Find an equation of the line sketched in Fig. 98. Check your equation with a graphing calculator.

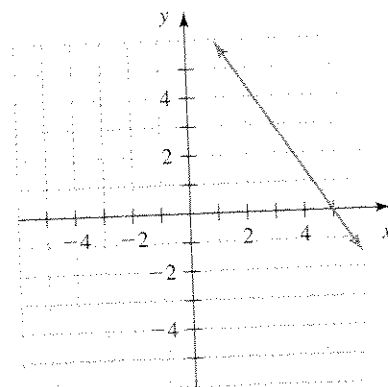


Figure 98 Exercise 73

- 74. Find an equation of the line sketched in Fig. 99. Check your equation with a graphing calculator.

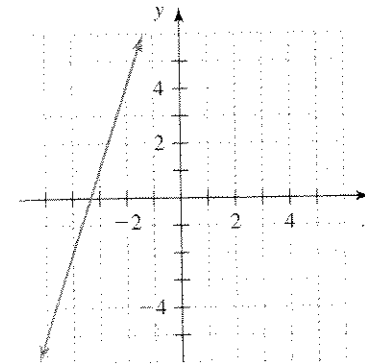


Figure 99 Exercise 74

- 75. Decide whether a line can have the indicated number of x -intercepts. If it is possible, find an equation of such a line. If it is not possible, explain why.
 - a. No x -intercepts
 - b. Exactly one x -intercept
 - c. Exactly two x -intercepts
 - d. An infinite number of x -intercepts
- 76. Decide whether a line can have the indicated number of y -intercepts. If it is possible, find an equation of such a line. If it is not possible, explain why.
 - a. No y -intercepts
 - b. Exactly one y -intercept
 - c. Exactly two y -intercepts
 - d. An infinite number of y -intercepts
- 77. Is there a line that contains all of the given points? If so, find an equation of it. If not, find an equation of a line that contains most of the given points.
 - $(-4, 15)$, $(-1, 9)$, $(3, 1)$, $(4, -1)$, $(9, -11)$
- 78. Is there a line that contains all of the given points? If so, find an equation of it. If not, find an equation of a line that contains most of the given points.
 - $(-3, 7)$, $(-1, 5)$, $(1, 1)$, $(3, -3)$, $(4, -5)$
- 79. Create a table of seven pairs of values of x and y for which
 - a. each point lies on the line $y = 3x - 6$.
 - b. each point lies close to, but not on, the line $y = 3x - 6$.

Table 24: Relationship Described by a Table

| x | y |
|-----|-----|
| 1 | 2 |
| 2 | 1 |
| 3 | 3 |
| 4 | 4 |

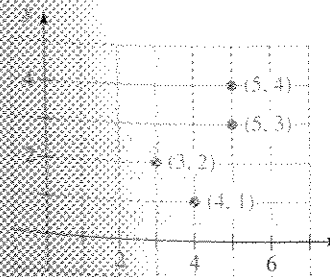


Figure 100 The relationship of Table 24 described by a graph

1.6 FUNCTIONS

Objectives

- ✦ Know the meanings of *relation*, *domain*, *range*, and *function*.
- ✦ Identify functions by using the *vertical line test*.
- ✦ Know the definition of a *linear function*.
- ✦ Know the Rule of Four for functions.
- ✦ Use the graph of a function to find the function's domain and range.

Throughout this chapter, we have described relationships between two variables. In this section, we will discuss how to describe some of these relationships by using an extremely important concept called a *function*.

Relation, Domain, Range, and Function

In this chapter we have used graphs, tables, and equations to describe the relationship between two variables. For example, Table 24 describes a relationship between the variables x and y . This relationship is also described graphically in Fig. 100.

- c. the points do not lie close to the line $y = 3x - 6$, but all of them lie close to another line. In addition to creating the table, provide an equation of the other line.
- 80. Suppose that a set of points all lie 0.5 unit above the line $y = -4x + 3$. Find an equation of the line that passes through the points of the set.
- 81. a. Find an equation of a line with slope -4 .
b. Find an equation of a line with y -intercept $(0, \frac{3}{7})$. Verify your result with a graphing calculator.
c. Find an equation of a line that contains the point $(-2, 8)$. Verify your result with a graphing calculator.
d. Determine whether there is a line that has slope -4 , has y -intercept $(0, \frac{3}{7})$, and contains the point $(-2, 8)$. Explain.
- 82. Find equations of two perpendicular lines that intersect at the point $(3, 1)$.
- 83. A student thinks that if a line has slope 2 and contains the point $(3, 5)$, then the equation of the line is $y = 2x + 5$, because the slope is 2 (the coefficient of x) and the y -coordinate of $(3, 5)$ is 5 (the constant term). What would you tell the student?
- 84. A student tries to find an equation of the line that contains the points $(1, 5)$ and $(3, 9)$. The student believes that an equation of the line is $y = 4x + 1$. The student then checks whether $(1, 5)$ satisfies $y = 4x + 1$:

$$y = 4x + 1$$

$$5 \stackrel{?}{=} 4(1) + 1$$

$$5 \stackrel{?}{=} 5$$

true

The student concludes that $y = 4x + 1$ is an equation of the line. Find any errors. Then find an equation correctly.
- 85. Describe how to find an equation of a line that contains two given points. How can you verify that the graph of the equation contains the two points?

We call the set of ordered pairs listed in Table 24 a *relation*. This relation consists of the ordered pairs (3, 2), (4, 1), (5, 3), and (5, 4). The *domain* of the relation is the set of all values of x (the independent variable)—in this case, 3, 4, and 5. The *range* of the relation is the set of all values of y (the dependent variable)—here, 1, 2, 3, and 4.

DEFINITION Relation, domain, and range

A **relation** is a set of ordered pairs. The **domain** of a relation is the set of all values of the independent variable, and the **range** of the relation is the set of all values of the dependent variable.

We can think of a relation as a machine in which values of x are “inputs” and values of y are “outputs.” In general, each member of the domain is an **input**, and each member of the range is an **output**.

For the relation described in Table 24, we can think of the values of x as being sent to the values of y (see Fig. 101).

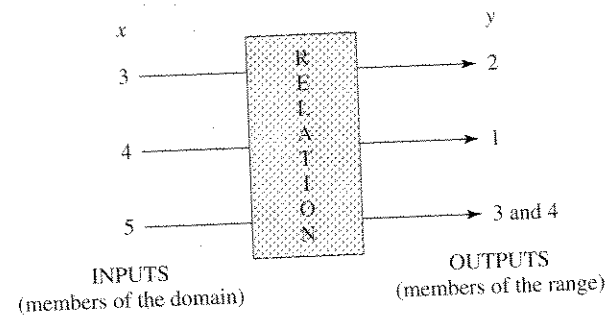


Figure 101 Think of a relation as an input–output machine

Note that the input $x = 5$ is sent to *two* outputs: $y = 3$ and $y = 4$. In a special type of relation called a *function*, each input is sent to exactly *one* output. The relation described in Table 24 is not a function.

DEFINITION Function

A **function** is a relation in which each input leads to exactly one output.

The equation $y = x + 2$ describes a relation consisting of an infinite number of ordered pairs. We will determine whether the relation is a function in Example 1.

Example 1 Deciding whether an Equation Describes a Function

Is the relation $y = x + 2$ a function? Find the domain and range of the relation.

Solution

Let’s consider some input–output pairs (in Fig. 102).

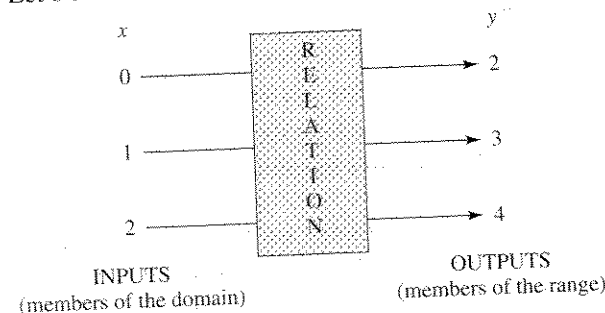


Figure 102 The “increasing by 2” relation: $y = x + 2$

Each input leads to just *one* output—namely, the input increased by 2—so the relation $y = x + 2$ is a function.

The domain of the relation $y = x + 2$ is the set of all real numbers, since we can add 2 to *any* real number. The range of $y = x + 2$ is also the set of real numbers, since any real number is the output of the number that is 2 units less than it.

Table 25 Input–Output Pairs of a Relation

| x (input) | y (output) |
|----------------|-----------------|
| 0 | 2 |
| 1 | 3 |
| 1 | 5 |
| 2 | 7 |
| 3 | 10 |

Example 2 Deciding whether an Equation Describes a Function

Is the relation $y = \pm x$ a function?

Solution

If $x = 1$, then $y = \pm 1$. So, the input $x = 1$ leads to *two* outputs: $y = -1$ and $y = 1$. Therefore, the relation $y = \pm x$ is not a function.

Example 3 Deciding whether an Equation Describes a Function

Is the relation $y^2 = x$ a function?

Solution

Let’s consider the input $x = 4$. We substitute 4 for x and solve for y :

$$y^2 = 4 \quad \text{Substitute 4 for } x.$$

$$y = -2 \quad \text{or} \quad y = 2 \quad (-2)^2 = 4, 2^2 = 4$$

The input $x = 4$ leads to *two* outputs: $y = -2$ and $y = 2$. So, the relation $y^2 = x$ is not a function.

Example 4 Deciding whether a Table Describes a Function

Is the relation described by Table 25 a function?

Solution

The input $x = 1$ leads to *two* outputs: $y = 3$ and $y = 5$. So the relation is not a function.

Example 5 Deciding whether a Graph Describes a Function

Is the relation described by the graph in Fig. 103 a function?

Solution

See Fig. 104. The input $x = 3$ leads to *two* outputs: $y = -4$ and $y = 4$. So, the relation is not a function.

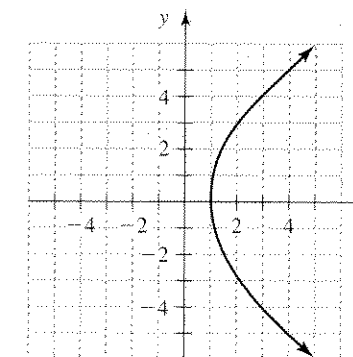


Figure 103 Graph of a relation

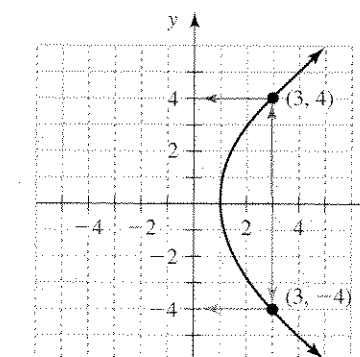


Figure 104 The input $x = 3$ gives two outputs: $y = -4$ and $y = 4$

Vertical Line Test

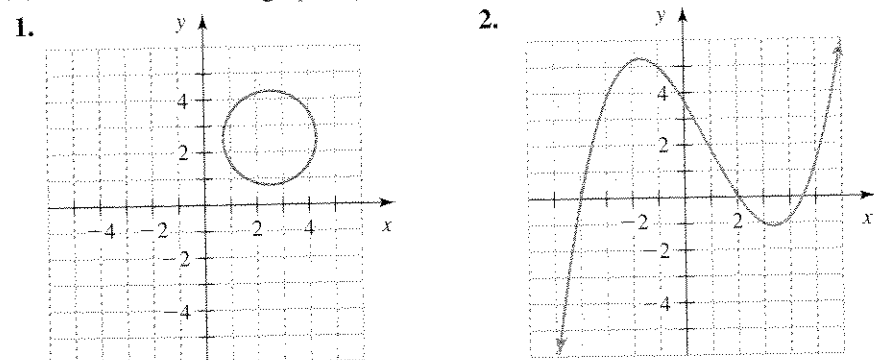
Notice that the relation described in Example 5 is not a function because some vertical lines would intersect the graph more than once.

Vertical Line Test

A relation is a function if and only if every vertical line intersects the graph of the relation at no more than one point. We call this requirement the **vertical line test**.

Example 6 Deciding whether a Graph Describes a Function

Determine whether the graph represents a function.



Solution

1. Since the vertical line sketched in Fig. 105 intersects the circle more than once, the relation is not a function.
2. Each vertical line sketched in Fig. 106 intersects the curve at just one point. In fact, any vertical line would intersect this curve at just one point. So, the relation is a function.

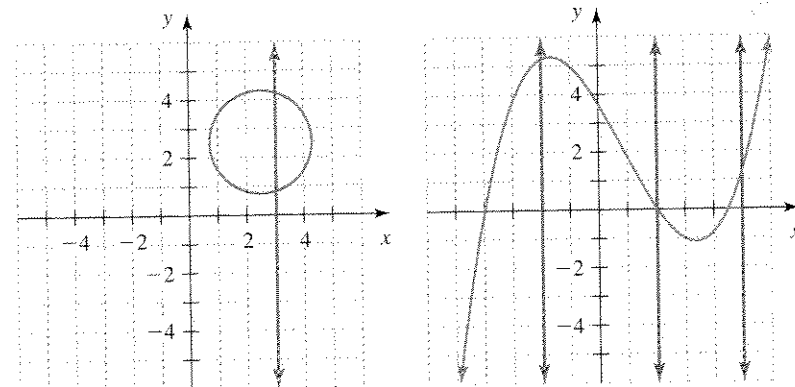


Figure 105 The circle does not describe a function

Figure 106 The curve describes a function

Example 7 Deciding whether an Equation Describes a Function

Is the relation $y = 2x + 1$ a function?

Solution

We begin by sketching the graph of $y = 2x + 1$ in Fig. 107. Note that each vertical line would intersect the line $y = 2x + 1$ at just one point. So, the relation $y = 2x + 1$ is a function.

Linear Functions

In Example 7, we saw that the line $y = 2x + 1$ is a function. In fact, any nonvertical line is a function, since it passes the vertical line test.

DEFINITION Linear function

A **linear function** is a relation whose equation can be put into the form

$$y = mx + b$$

where m and b are constants.

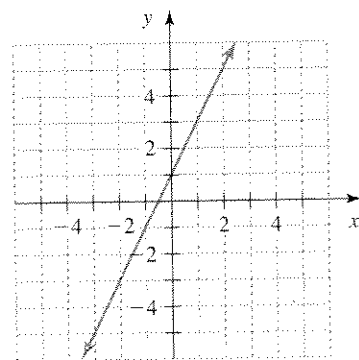


Figure 107 Graph of $y = 2x + 1$

Table 26 Input-Output Pairs for $y = 2x + 1$

| x | y |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |

Rule of Four for Functions

We can describe functions in four ways. For instance, in Example 7 we described the function $y = 2x + 1$ by using (1) the equation and (2) a graph (see Fig. 107). We can also describe some of the input-output pairs for the same function by using (3) a table (see Table 26). Finally, we can describe the function (4) verbally: In this case, for each input-output pair, the output is 1 more than twice the input.

Rule of Four for Functions

We can describe some or all of the input-output pairs of a function by means of

1. an equation,
2. a graph,
3. a table, or
4. words.

These four ways to describe input-output pairs of a function are known as the **Rule of Four** for functions.

Example 8 Describing a Function by Using the Rule of Four

1. Is the relation $y = -2x - 1$ a function?
2. List some input-output pairs of $y = -2x - 1$ by using a table.
3. Describe the input-output pairs of $y = -2x - 1$ by using a graph.
4. Describe the input-output pairs of $y = -2x - 1$ by using words.

Solution

1. Since $y = -2x - 1$ is of the form $y = mx + b$, it is a (linear) function.
2. We list five input-output pairs in Table 27.
3. We graph $y = -2x - 1$ in Fig. 108.
4. For each input-output pair, the output is 1 less than -2 times the input.

Table 27 Input-Output Pairs for $y = -2x - 1$

| x | y |
|----|------------------|
| -2 | $-2(-2) - 1 = 3$ |
| -1 | $-2(-1) - 1 = 1$ |
| 0 | $-2(0) - 1 = -1$ |
| 1 | $-2(1) - 1 = -3$ |
| 2 | $-2(2) - 1 = -5$ |

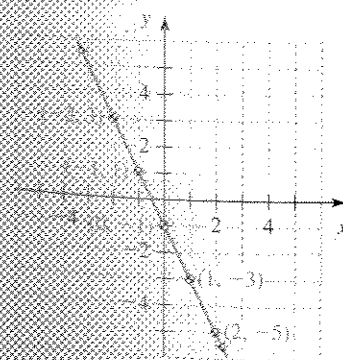


Figure 108 Graph of $y = -2x - 1$

Using a Graph to Find the Domain and Range of a Function

To describe the domain or range of a function, it is sometimes helpful to use the *inequality symbols* \leq and \geq . The symbol \leq means "is less than or equal to"; the symbol \geq means "is greater than or equal to." For example, the inequality $x \leq 4$ means that all values of x are less than or equal to 4. And the inequality $y \geq 7$ means that all values of y are greater than or equal to 7.

The inequality $5 \leq x$ means that 5 is *less* than or equal to all values of x . Notice that it is more natural to say that all values of x are *greater* than or equal to 5, which is true.

The inequality $2 \leq x \leq 6$ means that $2 \leq x$ and $x \leq 6$: All values of x are *both* greater than or equal to 2 *and* less than or equal to 6. In other words, all values of x are between 2 and 6, inclusive.

In this chapter, we have made many observations about linear equations. Since a linear function can be described by a linear equation, these observations tell us about linear functions. Let's summarize what we know about a linear function $y = mx + b$:

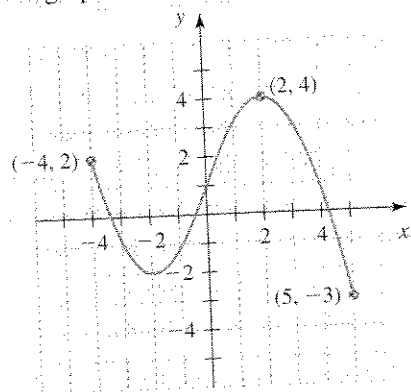
1. The graph of the function is a nonvertical line.
2. The constant m is the slope of the line, a measure of the line's steepness.
3. If $m > 0$, the graph of the function is an increasing line.
4. If $m < 0$, the graph of the function is a decreasing line.
5. If $m = 0$, the graph of the function is a horizontal line.
6. If an input increases by 1, then the corresponding output changes by the slope m .
7. If the run is 1, the rise is the slope m .
8. The y -intercept of the line is $(0, b)$.

Finally, since a linear equation of the form $y = mx + b$ is a *function*, we know that each input leads to exactly one output.

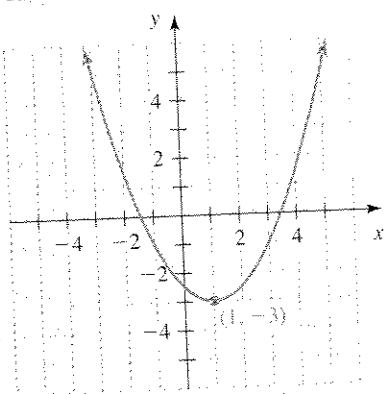
Example 9 Finding the Domain and Range

Use the graph of the function to determine the function's domain and range.

1.



2.



Solution

1. The domain is the set of all x -coordinates of points in the graph. Since there are no breaks in the graph, and since the leftmost point is $(-4, 2)$ and the rightmost point is $(5, -3)$, the domain is $-4 \leq x \leq 5$.

The range is the set of all y -coordinates of points in the graph. Since the lowest point is $(5, -3)$ and the highest point is $(2, 4)$, the range is $-3 \leq y \leq 4$.

2. The graph extends to the left and right indefinitely without breaks, so every real number is an x -coordinate of some point in the graph. The domain is the set of all real numbers.

The output -3 is the smallest number in the range, because $(1, -3)$ is the lowest point in the graph. The graph also extends upward indefinitely without breaks, so every number larger than -3 is also in the range. The range is $y \geq -3$.

group exploration

Vertical line test

1. Consider the relation described by Table 28. Is the relation a function? Explain. Now plot the points on a coordinate system. What do you notice about them?
2. Consider the relation described by Table 29. Is the relation a function? Explain. Now plot the points on a coordinate system. What do you notice about them?
3. Describe the graph of a relation that is not a function.
4. Determine whether each graph in Fig. 109 is the graph of a function. Explain.

Table 28 A Relation Described by a Table

| x | y |
|-----|-----|
| 2 | 1 |
| 2 | 5 |
| 2 | 7 |

Table 29 A Relation Described by a Table

| x | y |
|-----|-----|
| 4 | 2 |
| 4 | 3 |
| 4 | 6 |

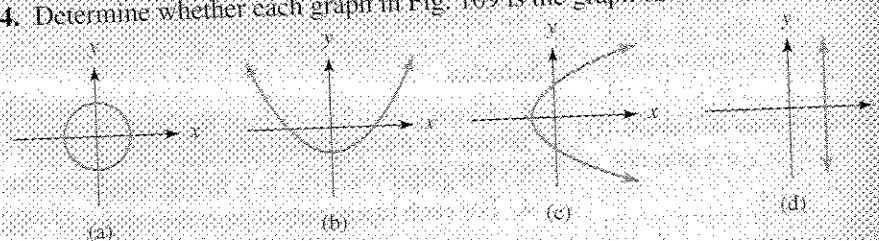


Figure 109 Which graphs describe functions?

Table 30 Camaro Production

| Year | Camaro Production (thousands) |
|------|-------------------------------|
| 1996 | 67 |
| 1997 | 56 |
| 1998 | 48 |
| 1999 | 41 |
| 2000 | 42 |
| 2001 | 29 |

Source: BlueOvalNews, Independent Voice of the Ford Community.

group exploration

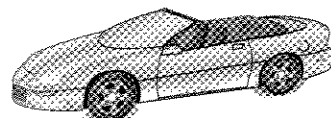
Looking ahead: Linear modeling

The production of the Chevrolet Camaro has decreased since 1996 (see Table 30).

1. Let C be Camaro production (in thousands), and let t be the number of years since 1995. For example, $t = 1$ represents 1996, because 1996 is 1 year since 1995. So, the 1996 production of 67 thousand Camaros is represented by $t = 1$ and $C = 67$. The information in Table 30 can be summarized with a table of values for t and C . Create such a table by filling in the missing entries in Table 31.

Table 31 Values of t and C for the Camaro Data

| Number of Years since 1995 | Camaro Production (in thousands) |
|----------------------------|----------------------------------|
| t | C |
| 1 | 67 |
| 2 | 56 |
| 3 | |
| 4 | 42 |
| | 29 |



2. Plot the points (t, C) that you listed in Table 31 on a coordinate system like the one in Fig. 110.

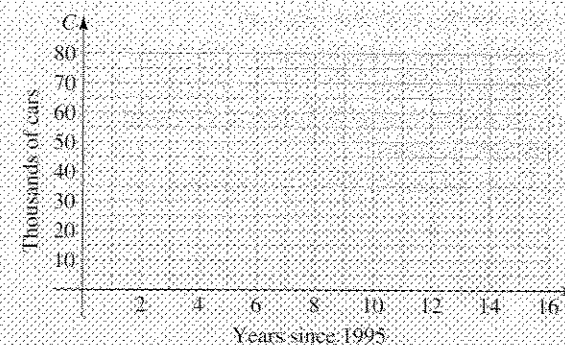


Figure 110 Plot the data points

3. When examining your graph, what do you notice about the arrangement of your plotted points?
4. Sketch a line that comes close to the six data points.
5. Use the line to estimate when 10 thousand Camaros were produced.
6. Use the line to estimate the number of Camaros produced in 2002.
7. Find the t -intercept of the line. What does the t -intercept mean in this situation? Will this prediction happen for certain? Explain.
8. Explain why it is not surprising that 2002 was the last year for Camaros to be manufactured.

TIPS FOR SUCCESS: Practice Exams

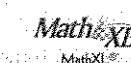
When studying for an exam (or a quiz), try creating your own exam to take for practice. Select several Homework exercises from each section on which you will be tested. Choose a variety of exercises that address concepts that your instructor has emphasized. Include many exercises that are moderately difficult and some that are challenging. Completing such a practice test will help you reflect on important concepts and pin down what types of problems you need to study more.

Work on the practice exam for a predetermined time period. Doing so will help you get used to a timed exam, build your confidence, and lower your anxiety for the real exam.

If you are studying with another student, each of you can create a test and then take each other's test. Or create a test together and each take it separately.

HOMEWORK 1.6

FOR EXTRA HELP



1. Some ordered pairs of four relations are listed in Table 32. Which of these relations could be functions? Explain.

Table 32 Which Relations Might Be Functions? (Exercise 1)

| Relation 1 | | Relation 2 | | Relation 3 | | Relation 4 | |
|------------|-----|------------|-----|------------|-----|------------|-----|
| x | y | x | y | x | y | x | y |
| 1 | 3 | 3 | 27 | 0 | 4 | 5 | 10 |
| 2 | 3 | 4 | 24 | 1 | 4 | 6 | 20 |
| 3 | 5 | 5 | 21 | 2 | 4 | 7 | 30 |
| 4 | 7 | 6 | 18 | 3 | 4 | 8 | 40 |
| 5 | 9 | 7 | 15 | 4 | 4 | 8 | 50 |

2. Some ordered pairs of four relations are listed in Table 33.
 - a. Which of the relations could be functions? Explain.
 - b. Which could be linear functions? Explain.

Table 33 Which Relations Might Be Functions? (Exercise 2)

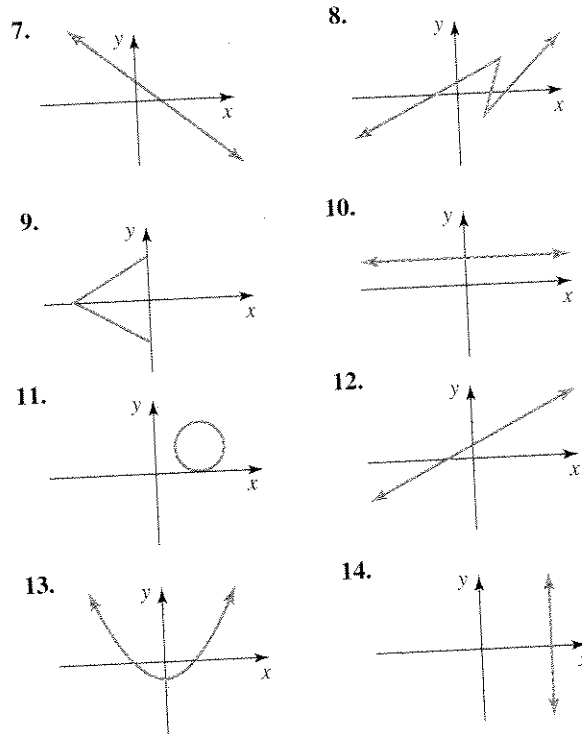
| Relation 1 | | Relation 2 | | Relation 3 | | Relation 4 | |
|------------|-----|------------|-----|------------|-----|------------|-----|
| x | y | x | y | x | y | x | y |
| 1 | 3 | 5 | 27 | 0 | 50 | 3 | 11 |
| 2 | 4 | 5 | 24 | 1 | 45 | 4 | 13 |
| 3 | 5 | 5 | 21 | 2 | 40 | 5 | 17 |
| 3 | 6 | 5 | 18 | 3 | 35 | 6 | 25 |
| 4 | 7 | 5 | 15 | 4 | 30 | 7 | 40 |

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3. For a certain relation, an input leads to two different outputs. Could the relation be a function? Explain.
4. For a certain relation, two different inputs lead to the same output. Could the relation be a function? Explain.
5. A relation's graph contains the points (2, 3) and (5, 3). Could the relation be a function? Explain.
6. A relation's graph contains the points (4, 5) and (4, 9). Could the relation be a function? Explain.

Use the graph of the function to determine the function's domain and range.

Determine whether the graph represents a function. Explain.



Determine whether the relation is a function. Explain.

15. $y = 5x - 1$
16. $y = -3x + 8$
17. $2x - 5y = 10$
18. $4x + 3y = 24$
19. $y = 4$
20. $y = -1$
21. $x = -3$
22. $x = 0$

25. Is a nonvertical line the graph of a function? Explain.

26. Is a vertical line the graph of a function? Explain.

27. Is a circle the graph of a function? Explain.

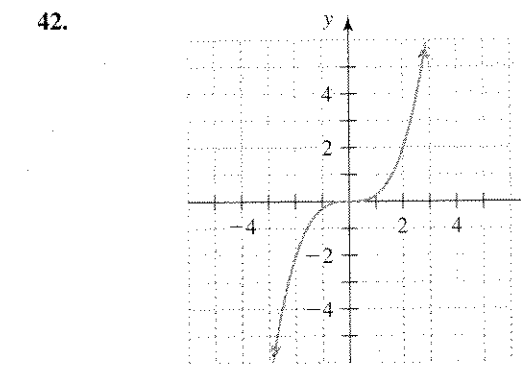
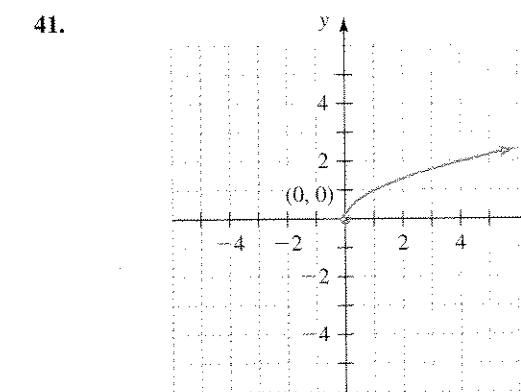
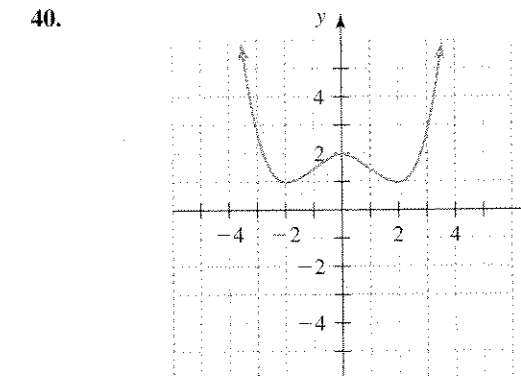
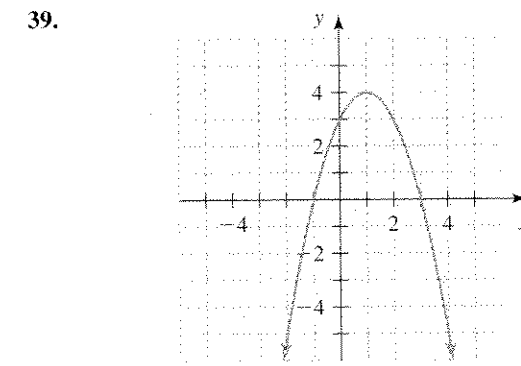
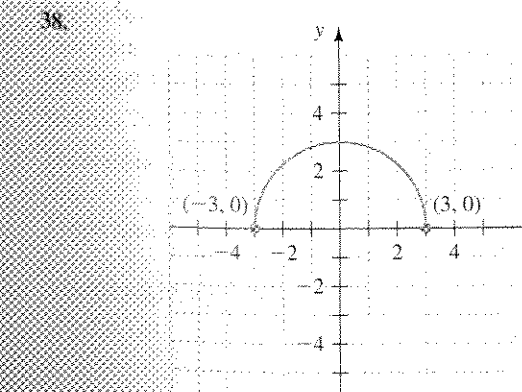
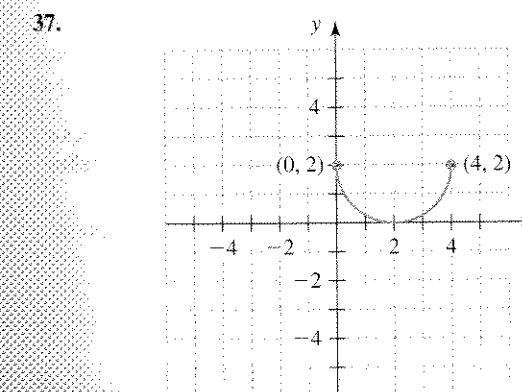
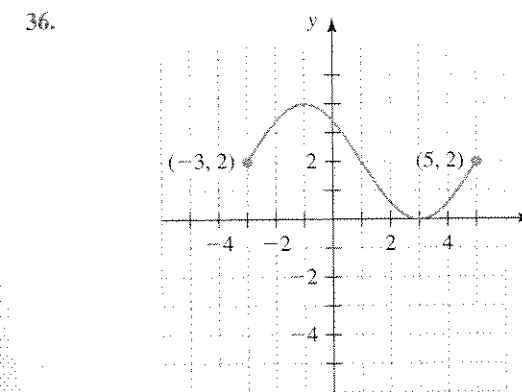
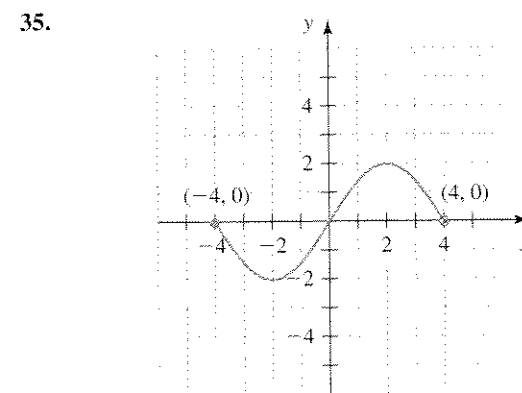
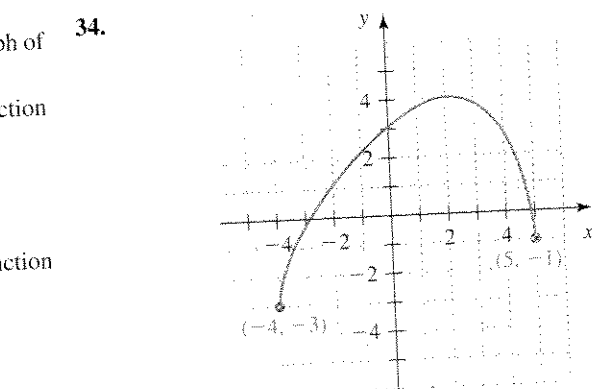
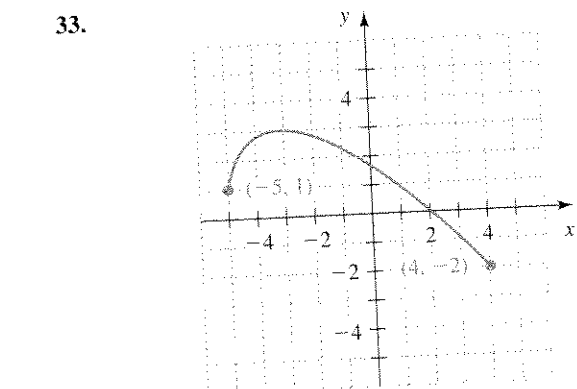
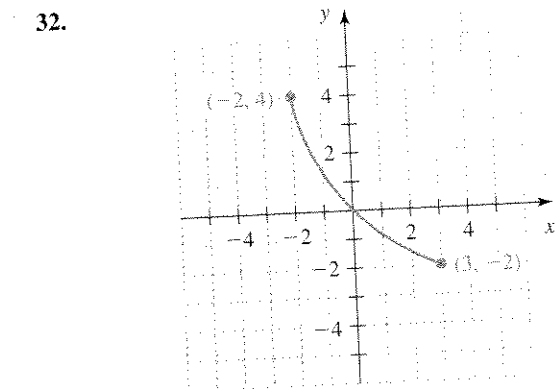
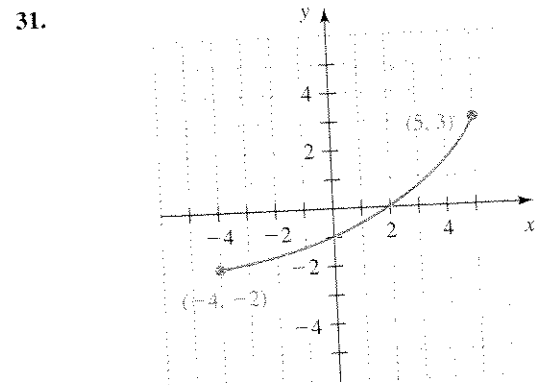
28. Is a semicircle that is the "upper half" of a circle the graph of a function? Explain.

29. Describe the Rule of Four as applied to the function $y = 3x - 2$:

- a. Describe five input-output pairs by using a table.
- b. Describe the input-output pairs by using a graph.
- c. Describe the input-output pairs by using words.

30. Describe the Rule of Four as applied to the function $y = \frac{1}{2}x + 2$:

- a. Describe five input-output pairs by using a table.
- b. Describe the input-output pairs by using a graph.
- c. Describe the input-output pairs by using words.



43. Describe the input-output pairs of a function (different from those in this section) by using an equation, a graph, and words. Describe also five input-output pairs of the function by using a table. Explain why your relation is a function.

44. Sketch the graph of a relation (different from those in this section) that is not a function. Next, create a table that lists five ordered pairs of the relation. Explain why your relation is not a function.

