## Math 110

Function Notes

Show all relevant work!

We've seen a number of examples in algebra where two (or more) quantities are related through a formula, graph, or table. We typically look at these relationships as a connection between an input value and a resulting output. In most cases we might even go so far as to distinguish one quantity as dependent on the other. For example, when you hire a taxi, the cost of the ride is *dependent* on the distance you travel. Consider the examples graphed below where the height of a flag changes with (depends on) time.

1. Lamont is hoisting a flag up (or down) a flag pole. Each graph below models a different way in which this activity might take place. For each graph, write a description of the way (speed and direction) the flag's height changes over time.





Notice that all of the graphs describe a realistic (or at least possible) situation except one. Because we typically use mathematics to model reality, it would be good if we avoided situations that are inherently flawed. For the reason we restrict relationships between numbers so that it isn't possible to have a case like (g) where something can be in more than one place at the same time.

## Notation:

Function notation is an equivalent way of writing an equation where we replace the dependent (output) variable, usually y, with the symbol f(x) (or g(x), h(x) etc.). For example,  $y = 3x^2 - 2x$  is written  $f(x) = 3x^2 - 2x$  in function notation. The change in notation gives us a shortcut in evaluating functions:

Instead of saying, Evaluate  $y = 3x^2 - 2x$  for x = -4, we can write, If  $f(x) = 3x^2 - 2x$ , find f(-4). The symbol f(x) can be read, "f at x," or "f of x." Then f(3) means the value of the function, f, at the input value of x = 3.

Another benefit of using this notation is that it allows us to differentiate between two functions. Instead of y = 3x + 7 and y = -2x + 5, we can avoid confusion by writing f(x) = 3x + 7 for the first function and g(x) = -2 + 5 for the second.

The graph to the right shows how x and g(x) are related – just like x and y.

2. Use the exponential function, g(x), graphed below to answer the given questions.

- (a) Estimate g(2).
- (b) Estimate x where g(x) = 5.
- (c) Estimate the solution to g(x) = x.





3. The following table shows values for the function f.



4. The table shows the cost, C(m), of a taxi ride as a function of the number of miles, m, traveled.

m	0	1	2	3	4	5
C(m)	0	2.50	4.00	5.50	7.00	8.50

(a) Estimate and interpret C(3.5) in practical terms.

(b) Estimate and interpret the solution to C(m) = 3.5 in practical terms.

5. A ball is thrown straight up and its height above the ground (in feet) is given as a function of time (in seconds) by  $h(t) = -16t^2 + 50t + 6$ .

(a) Find h(2) and interpret its meaning.

- (b) Find h(3) and interpret its meaning.
- (c) Find h(4) and interpret its meaning.
- 6. A tub drains so that after 5 minutes 30 gallons of water remain in the tub and after 20 minutes, 21 gallons remain. (a) If the amount of water in the tub is linearly dependent on the time since the drain was opened, write the equation
  - of the volume of water in the tub, V, as a function of time, t.
  - (b) What does the slope of this line tell you?
  - (c) How much water is there in the tub when it begins draining?
  - (d) Find the *t*-intercept and interpret its meaning in this context.

- 7. Norbert's velocity, v(t) (in ft/sec) as he rides his bike away from home is graphed as a function of time (in seconds). Assume that positive velocity is in the direction away from home and negative velocity is towards home.
  - (a) Find v(5) and interpet its meaning.
  - (b) When is Norbert the farthest from home?
  - (c) At what times is Norbert stopped?

