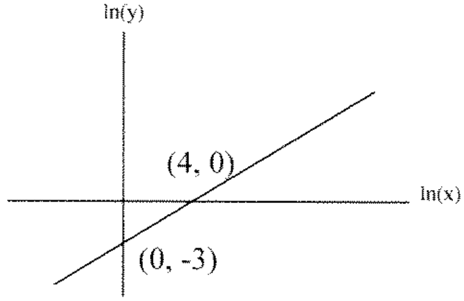
Show all appropriate work for full credit.

1. Given the graph below, find a formula for y in terms of x.



$\ln y = \frac{3}{4} \ln x - 3$

$y = e^{\ln x^{\frac{3}{4}} - 3}$

$y = e^{\ln x^{\frac{3}{4}}} \cdot e^{-3}$

$y = x^{\frac{3}{4}} \cdot e^{-3}$

2. Fill in all the blanks in the table for which you have sufficient information.

x	-3	-2	-2	0	1	2	3
f(x)	4	2	-2	-3	0	4	3
f(-x)	3	4	0	-3	-2	2	4
-f(x)	-4	-2	2	3	0	-4	-3
f(x+3)	-3	0	4	3			
f(x)+3	7	5	1	0	3	7	6

3. Let $g(x) = \sqrt{4 - x^2}$.
   a. Write a function that will translate the graph of g two units to the right and one unit up.
   b. Give the domain and range of the new function.

Three units to the right: $h(x) = g(x - 2)$

Therefore, $h(x) = \sqrt{4 - (x - 2)^2}$

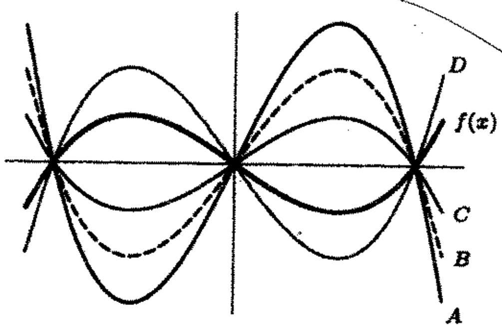
Then, shift one unit up: $h(x) = \sqrt{4 - (x - 2)^2} + 1$

Domain: $4 - (x - 2)^2 > 0, \therefore 0 \le x \le 4$

Range: $1 \le y \le 3$

4. The following figure shows the graph o $f(x)$f and the graphs of several multiples of $f(x)$.

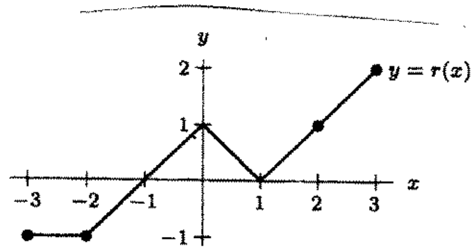
Suppose A is graph of $y = a \cdot f(x)$.
B is graph of $y = b \cdot f(x)$.
C is graph of $y = c \cdot f(x)$.
D is graph of $y = d \cdot f(x)$.



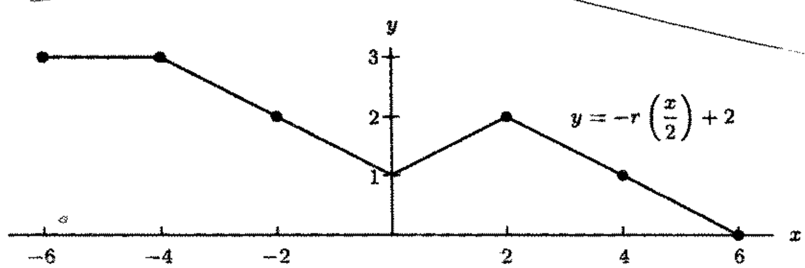
Circle the statements below which are true:

- $a < b$
- $a < 0$
- $b < c$
- $c < d$
- $c < 0$
- $d < 0$
- $d < 1$

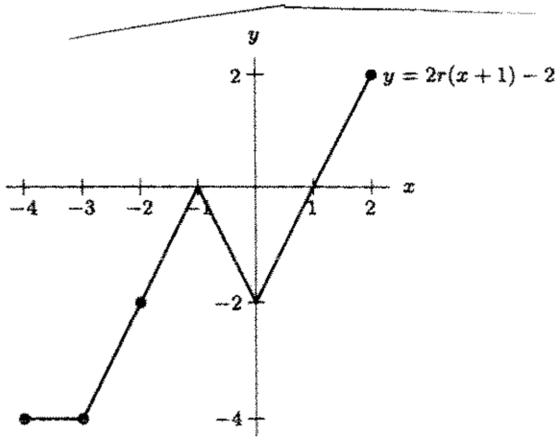
5. The graph of $y = r(x)$ is given.



a. Sketch $y = 2 - r(\frac{x}{2})$. Label both axes.



b. Sketch $y = 2r(x + 1) - 2$. Label both axes.



6. A projectile's height $h(t)$ in feet above ground is a quadratic function of time t in seconds since launched. Three values of the function are given in the table below:

t	$h(t)$
0	64
2	96
3	64

a. What is the practical interpretation of $h(0)$?

$h(0)$ is the height of the projectile above ground at the instant it is fired.

b. Find a formula for $h(t)$.

$$h(t) = at^2 + bt + c$$

$$\begin{cases} h(0) = 64 = a(0)^2 + b(0) + c \\ h(2) = 96 = a(2)^2 + b(2) + c \\ h(3) = 64 = a(3)^2 + b(3) + c \end{cases}$$

$$\begin{cases} c = 64 \\ 16 = 2a + b \\ 0 = 9a + 3b \end{cases}$$

$$\begin{cases} a = -16 \\ b = 48 \\ c = 64 \end{cases}$$

$$h(t) = -16t^2 + 48t + 64$$

c. Find the vertex of the parabola.

$$-\frac{b}{2a} = \frac{3}{2}$$

$$f\left(-\frac{b}{2a}\right) = 100$$

d. What is the physical interpretation of the vertex in the context of this problem?

The vertex is the highest point above ground to which the projectile rises.

7. Solve x in $\frac{\ln(\sqrt{x+4}+2)}{\ln\sqrt{x}} = 2$

$\ln(\sqrt{x+4} + 2) = 2\ln\sqrt{x}$

$\ln(\sqrt{x+4} + 2) = \ln x$

$\sqrt{x+4} + 2 = x$

$\sqrt{x+4} = x - 2 \quad x \geq 2$

$x + 4 = x^2 - 4x + 4$

$x^2 - 5x = 0$

~~$x = 0$~~ or $x = 5$

$\therefore \boxed{x = 5}$

8. Solve x in $(\pi x)^{\log_{10}\pi} = (ex)^{\log_{10} e}$

$\log(\pi x)^{\log\pi} = \log(ex)^{\log e}$

$\log\pi \cdot \log(\pi x) = \log e \cdot \log(ex)$

$\log\pi \cdot [\log(\pi) + \log x] = \log e \cdot [\log e + \log x]$

$(\log\pi)^2 + \log\pi\log x = (\log e)^2 + \log e \log x$

$\log\pi\log x - \log e\log x = (\log e)^2 - (\log\pi)^2$

$\log x(\log\pi - \log e) = (\log e)^2 - (\log\pi)^2$

$\log x = \frac{(\log e + \log\pi)(\log e - \log\pi)}{\log\pi - \log e}$

$\log x = -\log e - \log\pi$

$\log x = -\log\pi\cdot e$

$\log x = \log\frac{1}{\pi e}$

$\boxed{x = \frac{1}{\pi\cdot e}}$

9. Let $f(x) = \ln(x + \sqrt{x^2+1})$. Find $f^{-1}(x)$.

$y = \ln(x + \sqrt{x^2+1})$

$e^y = x + \sqrt{x^2+1}$

$\sqrt{x^2+1} = e^y - x$

~~x^2~~ $+1 = e^{2y} - x\cdot e^y$ ~~$+x^2$~~

$xe^y = e^{2y} - 1$

$x = \frac{e^{2y}-1}{e^y}$

$y = \frac{e^{2x}-1}{e^x}$

$\therefore \boxed{f^{-1}(x) = \frac{e^{2x}-1}{e^x}}$

10. Suppose that $\log 2 = a$, $\log 3 = b$. Solve for x in terms of a and b:

$$6^x = \frac{10}{3} - 6^{-x}$$

~~$6^x - 6^{-x} = \frac{10}{3}$~~

$3\cdot 6^x = 10 - 3\cdot 6^{-x}$

$3\cdot 6^x - 10 + 3\cdot 6^{-x} = 0$

let $6^x = y^2$, $6^{-x} = y^{-2}$

$3y^2 - 10 + 3y^{-2} = 0$

$(3y^2 - y^{-2})(y^2 - 3y^{-2}) = 0$

$\therefore 3\cdot 6^x - 6^{-x} = 0$ or $6^x - 3\cdot 6^{-x} = 0$

$3\cdot 6^x = 6^{-x}$ $\quad$ $6^x = 3\cdot 6^{-x}$

$\log 3 + x\cdot\log 6 = -x\log 6$ $\quad$ $x\log 6 = \log 3 - x\log 6$

$x = \frac{\log 3}{-2\log 6} = \frac{\log 3}{-2(\log 2 + \log 3)} = \boxed{\frac{b}{-2(a+b)}}$ $\quad$ $x = \frac{\log 3}{2\log 6} = \frac{\log 3}{2(\log 2 + \log 3)} = \boxed{\frac{b}{2(a+b)}}$