Show all appropriate work.

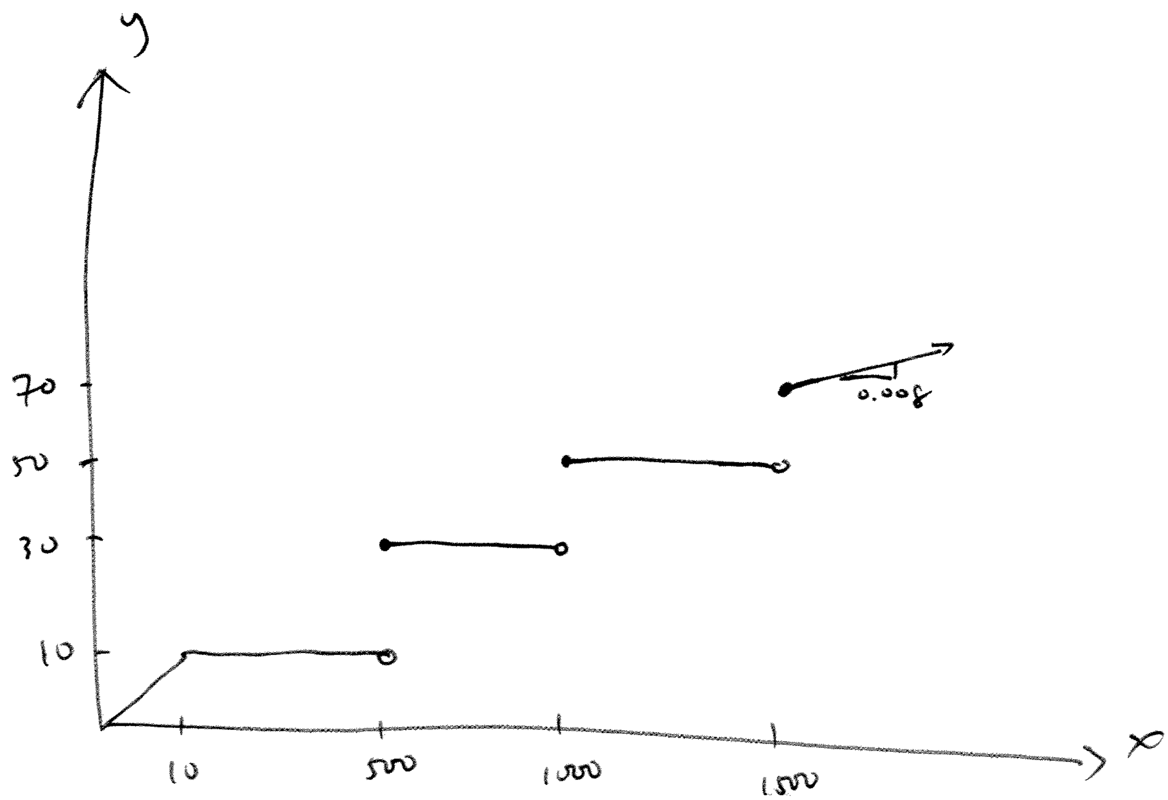
1. Holders of credit cards issued by banks, department stores, oil companies, and so on, receive bills each month that state minimum amounts that must be paid by a certain due date. The minimum due depends on the total amount owed. Once such credit card company uses the flowing rules: For a bill of less than $10, the entire amount is due. For a bill of at least $10 but less than $500, the minimum due is $10. a minimum of $30 is due on a bill of at least $500 but less than $1000, a minimum of $50 is due on a bill of at least $1000 but less than $1500, and a minimum of $70 plus 0.8% of additional spending on the top of $1500 is due on bills of $1500 or more.

A. Find the function f that describes the minimum payment due on a bill of x dollars. Let y be the minimum payment.

x be the bill.

$$y = \begin{cases} x & 0 \le x < 10 \\ 10 & 10 \le x < 500 \\ 30 & 500 \le x < 1000 \\ 50 & 1000 \le x < 1500 \\ 70 + 0.008(x - 1500) & x \ge 1500 \end{cases}$$

B. Sketch a graph f.



2. The size of a colony of bacteria is decreasing exponentially. At the end of 4 hours there are 5000 bacteria. At the end of 7 hours there are 2000.

A. Write a formula for the population of bacteria at time t, in hours.

$y = ab^t$

$$\frac{5000 = a \cdot b^4}{2000 = a \cdot b^7}$$

$\frac{5}{2} = b^{-3}$

$b^3 = \frac{2}{5}$

$b = \sqrt[3]{\frac{2}{5}} \approx 0.74$

$\therefore 5000 = a \cdot (0.74)^4$

$a = \frac{5000}{0.74^4} \approx 16674$

$\therefore \underline{y = 16674(0.74)^t}$

B. By what percent does the number of bacteria decrease each hour?

$b = 1 + r$

$\cancel{r = 1 - b = t}$

$r = b - 1 = 0.74 - 1 = 0.26 \quad \therefore 26\%$

3. A person wishes to invest \$6000 into Account(1) paying 5.5% compounded continuously and \$3500 into Account(2) that pays 8.5% compounded daily.

A. Set up the two compounding formulas for each account.

Account(1)

$y = 6000e^{0.055t}$

Account(2)

$y = 3500\left(1 + \frac{0.085}{365}\right)^{365t}$

B. What is the effective rate of each account?

Account(1)

$y = 6000\left(e^{0.055}\right)^t$

$y = 6000(1.0565)^t$

$\therefore r = b - 1 = 1.0565 - 1 = 5.65\%$

Account(2)

$y = 3500\left[\left(1 + \frac{0.085}{365}\right)^{365}\right]^t$

$y = 3500(1.0887)^t$

$\therefore r = b - 1 = 1.0887 - 1 = 8.87\%$

C. When will the Account(2) be worth more than Account(1).

$y_1 = y_2$ where the worth the same.

$\therefore 6000(1.0565)^t = 3500(1.0887)^t$

$\frac{6000}{3500} = \left(\frac{1.0887}{1.0565}\right)^t$

$\ln\left(\frac{6500}{3500}\right) = t \ln\left(\frac{1.0887}{1.0565}\right)$

$t = \frac{\ln\left(\frac{6500}{3500}\right)}{\ln\left(\frac{1.0887}{1.0565}\right)} = 20.62$ yrs

4. On the axes below, sketch the graph of a continuous function, $y = f(x)$ with all of the following features.

A. $f(-5) = 0$

B. $f(-2) = 2$

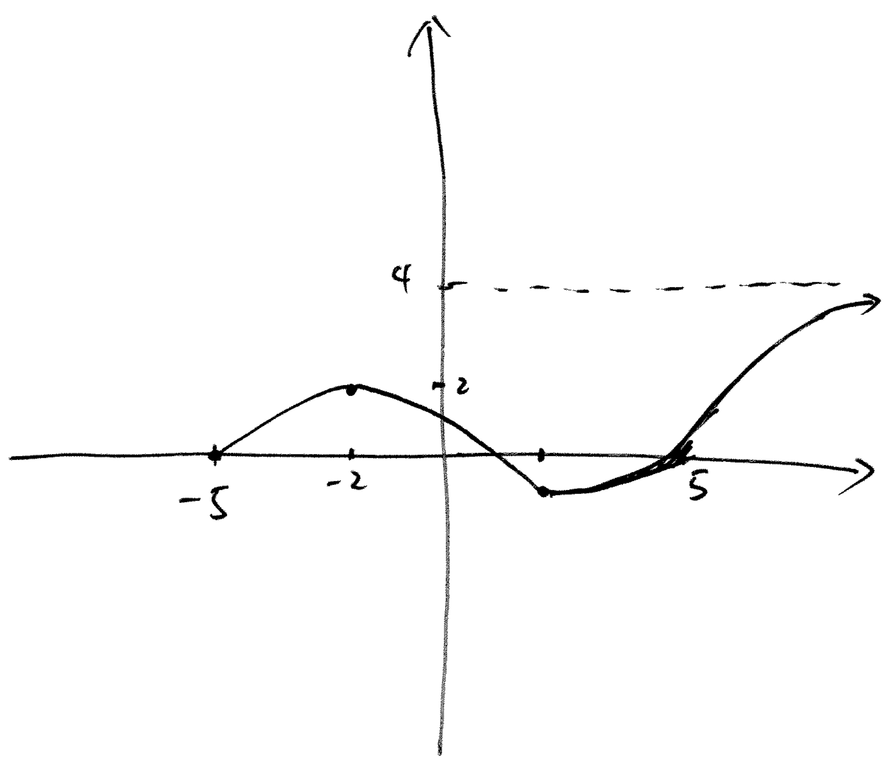
C. $f(2) = -1$

D. f is concave down for $x < 2$

E. f is concave up and increasing for $2 < x < 5$

F. f is concave down for $x > 5$.

G. $f \to 4$ for $x > 5$



$y = -3x^2 + 9x + 7$

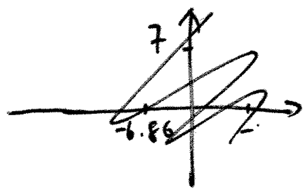
5. Solve and graph ~~$-3x^2+7=-9x$~~. (Mark the vertex, zeros, and y-int)

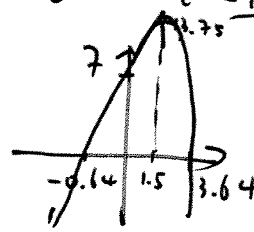
$h = -\frac{b}{2a} = -\frac{9}{-6} = \frac{3}{2}$. $k = -3\left(\frac{3}{2}\right)^2 + 9\left(\frac{3}{2}\right) + 7 = 13.75$

y-int : $(0, 7)$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-9 \pm \sqrt{81+84}}{2(-3)} = \frac{-9 \pm \sqrt{165}}{-6} = \begin{cases} \frac{-9+\sqrt{165}}{-6} = \cancel{-1.14} \ -0.64 \\ \frac{-9-\sqrt{165}}{-6} = \cancel{-6.86} \ 3.64 \end{cases}$

$\therefore$





6. The volume in cube ft. of a sphere of radius r is given by $A = f(r) = \frac{4}{3}\pi r^3$.

A. Find a formula for the inverse, f^{-1}.

$f(r) = \frac{4}{3}\pi r^3$

$y = \frac{4}{3}\pi r^3$

$r^3 = \frac{3y}{4\pi}$

$r = \sqrt[3]{\frac{3y}{4\pi}}$

$f^{-1}(r) = \sqrt[3]{\frac{3r}{4\pi}}$ //

B. Interpret the meaning of the inverse.

The radius in ft of a sphere of volume. $f^{-1}(r) = \sqrt[3]{\frac{3r}{4\pi}}$ //

C. What are the units of the inverse.

feet //

D. Evaluate $f^{-1}(r) = 256\pi$

$f^{-1}(r) = \sqrt[3]{\frac{3 \cdot 256\pi}{4\pi}} = \sqrt[3]{3 \cdot 64} = 4\sqrt[3]{3}$ ft //

7. One of the function is the table is linear, one is exponential, and the other is neither. Find formulas for the linear and exponential functions.

(i)

X	0.21	0.37	0.41	0.62	0.68
y	0.03193	0.04681	0.05053	0.07006	0.07564

(ii)

X	0.21	0.37	0.41	0.62	0.68
Y	3.324896	3.423316	3.448373	3.582963	3.622373

(iii)

X	0.21	0.37	0.41	0.62	0.68
Y	4.26584	4.24589	4.86321	5.86512	7.58946

iii) $\searrow$ then $\nearrow$ $\therefore$ neither.

i) $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0.04681 - 0.03193}{0.37 - 0.21} = 0.93$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0.05053 - 0.04681}{0.41 - 0.37} = 0.93$

$\therefore$ constant slope $\therefore$ linear.

$y - 0.03193 = 0.93(x - 0.21)$

$y = 0.93x - 0.93 \cdot 0.21 + 0.03193$

$y = 0.93x - 0.16337$ //.

ii) $\dfrac{y_1 = ab^{t_1}}{y_2 = ab^{t_2}}$ $\dfrac{3.423316}{3.324896} = b^{0.37-0.21}$ $b =$ ~~0.23079~~ 5.09×10^{-7}

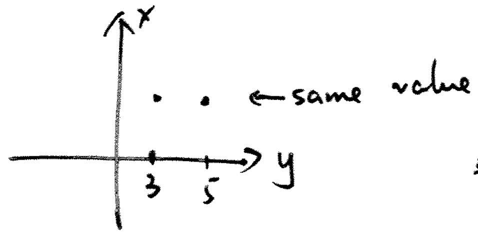
$\dfrac{y_1 = ab^{t_1}}{y_2 = ab^{t_2}}$. $\dfrac{3.448373}{3.423316} = b^{0.41-0.37}$. $b =$ ~~F12~~ small ~~enough~~ enough.

$\therefore$ $a = \dfrac{y}{b^t} = \dfrac{3.324896}{(5.09 \times 10^{-7})^{0.21}} = 69.72$

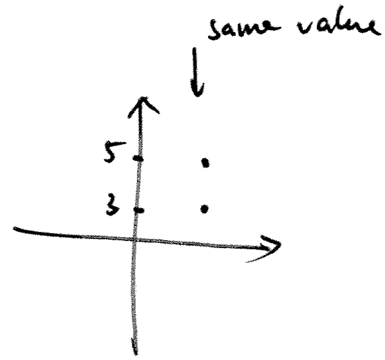
$\therefore$ $y = 69.72 (5.09 \times 10^{-7})^{t}$ //

True or False

8. If $g^{-1}(3) = g^{-1}(5)$, then g^{-1} is not invertible.



$g^{-1}(x)$



$g(x)$

$\therefore$ Does not pass vertical line test $\therefore$ True

9. Andy's girlfriend has \$1,000,000 in Andy's bank account. What is the dollar amount of this one million dollars after 50 years if the account has an interest rate of 8% **annually** (Andy owns the bank)? Repeat for compound **quarterly**, **daily**, and **continuously**.

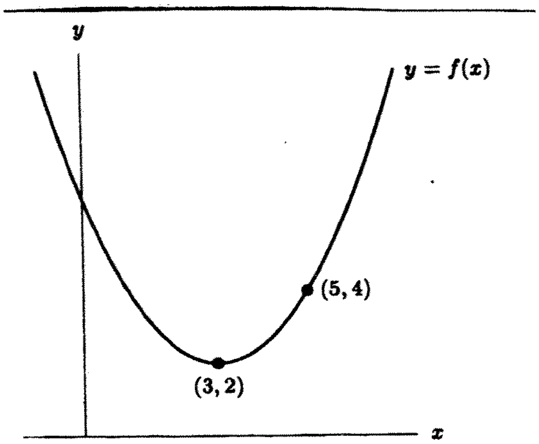
Annual: $y = 1(1 + 0.08)^{50} = \$46.9$ Million

Quarterly: $y = 1\left(1 + \frac{0.08}{4}\right)^{50 \cdot 4} = \$52.48$ Million

Daily: $y = 1\left(1 + \frac{0.08}{365}\right)^{365 \cdot 50} = \$54.57$ Million

Continuously: $y = 1e^{0.08t} = 1e^{0.08 \cdot 50} = \$54.598$ Million

10. Obtain the function from the following figures:



$y = a(x-h)^2 + k$

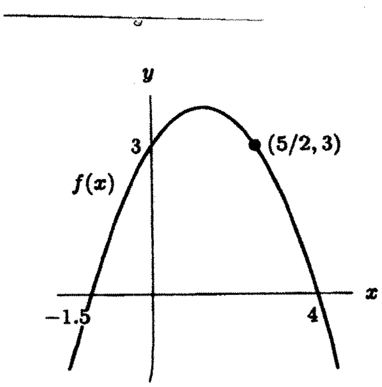
$y = a(x-3)^2 + 2$

$4 = a(5-3)^2 + 2$

$4 = 4a + 2$

$a = \frac{1}{2}$

$\therefore \quad y = \frac{1}{2}(x-3)^2 + 2$ //



$y = a(x - x_1)(x - x_2)$

$y = a(x+1.5)(x-4)$

$3 = a(1.5)(-4)$

$a = -\frac{1}{2}$

$\therefore \; y = -\frac{1}{2}(x+1.5)(x-4)$

To test, plug $x = \frac{5}{2}$ into the equation

$y = -\frac{1}{2}\left(\frac{5}{2}+1.5\right)\left(\frac{5}{2}-4\right) = 3$

$\therefore \left(\frac{5}{2}, 3\right)$ pass.

$\therefore \quad y = -\frac{1}{2}(x+1.5)(x-4)$ //.