MATH 130 Exam Four

Instructions: Please use pencil for this test. Use the test paper only to answer the exam. Label all calculations.

I certify that I have not nor will I discuss this test with other students until after I received my graded test. I have followed the rules for academic integrity as stated in the college catalog:

I will not copy from another student;
I will not allow another student to copy from me; and
I will not use any notes or books other than the 4x6 card allowed.

I certify that this examination is my own work.

Print Name: Solution

Signature: _____

Date: _____

GRADE _____

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

Show all appropriate work

1. Establish this identity: $\dfrac{\sin(2\theta)+\sin(4\theta)}{\sin(2\theta)-\sin(4\theta)}+\dfrac{\tan(3\theta)}{\tan\theta}=0$

$$LHS = \frac{2\sin\frac{2\theta+4\theta}{2}\cos\frac{2\theta-4\theta}{2}}{2\cos\frac{2\theta+4\theta}{2}\sin\frac{2\theta-4\theta}{2}} + \frac{\tan 3\theta}{\tan\theta}$$

$$= \frac{\cancel{2}\sin 3\theta\cos\theta}{\cancel{2}\cos 3\theta\sin(-\theta)} + \frac{\tan 3\theta}{\tan\theta}$$

$$= \frac{\tan(3\theta)}{-\tan\theta} + \frac{\tan 3\theta}{\tan\theta}$$

$$= 0 = RHS$$

2. Establish this identity: $(1+\cos\theta)\tan\dfrac{\theta}{2}=\sin\theta$

$$LHS = (\cancel{1+\cos\theta})\cdot\frac{\sin\theta}{\cancel{1+\cos\theta}}$$

$$= \sin\theta$$

3. 4. If $\cos\alpha = \frac{4}{5}, 0 < \alpha < \frac{\pi}{2}; \cos\beta = \frac{5}{13}, -\frac{\pi}{2} < \beta < 0$, please find the exact value of:

$$\tan(\alpha+\beta) \text{ and } \cos\frac{\alpha}{2}$$

$\cos\alpha = \frac{4}{5} = \frac{x}{r}$ $x=4$, $r=5$, $y=3$, QI.

$\cos\beta = \frac{5}{13} = \frac{x}{r}$ $x=5$, $y=-12$, $r=13$, QIV

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$= \frac{\frac{3}{4} + \frac{-12}{5}}{1 - \frac{3}{4}\cdot\left(\frac{-12}{5}\right)} = \left(\frac{\frac{3}{4} - \frac{12}{5}}{1 + \frac{36}{20}}\right)\cdot 20 = \frac{15 - 48}{20 + 36}$$

$$= \frac{-33}{56}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} = \pm\sqrt{\frac{1+\frac{4}{5}}{2}}$$

$$= \pm\sqrt{\frac{1}{2} + \frac{4}{10}}$$

$$= \pm\sqrt{\frac{9}{10}} \quad \text{QI}$$

$$= \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}.$$

5. Find the exact value of this expression: $\tan\left(2\sin^{-1}\frac{6}{11}\right)$

$= 2\tan\left(\sin^{-1}\frac{6}{11}\right)$

$1 - \tan^2\left(\sin^{-1}\frac{6}{11}\right)$

$\sin^{-1}\frac{6}{11} = \theta$

$\sin\theta = \frac{6}{11}$ QI

$y = 6$, $r = 11$

$x = \sqrt{11^2 - 6^2} = \sqrt{121 - 36}$

$= \sqrt{85}$

$\tan\theta = \frac{6}{\sqrt{85}}$

$= \frac{2 \cdot \frac{6}{\sqrt{85}}}{1 - \frac{36}{85}} = \frac{\frac{12\sqrt{85}}{85}}{1 - \frac{36}{85}}$

$= \frac{12\sqrt{85}}{85 - 36} = \frac{12\sqrt{85}}{49}$

6. Find the exact value of this expression: $\sin\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$

$\cos^{-1}\frac{3}{5} = \theta$ QI.

$\cos\theta = \frac{3}{5}$ $x = 3$

$r = 5$

$y = 4$

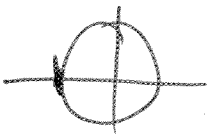
$= \pm\sqrt{\frac{1 - \cos x}{2}}$

$= \sqrt{\frac{1 - \cos x}{2}} = \sqrt{\frac{1 - \frac{3}{5}}{2}}$

$= \sqrt{\frac{\frac{2}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$

7. Solve this equation on the interval $0 \le \theta \le 2\pi$: $\sin\theta = \tan\theta$

$\sin\theta = \frac{\sin\theta}{\cos\theta}$



$\sin\theta \cdot \cos\theta = \sin\theta$

$\cos\theta = 1 = \frac{x}{r}$

$\theta = \pi + 2k\pi$

$\theta = \pi$.

8. Find the exact value of this expression: $\sin\left(\cos^{-1}\frac{5}{13}-\cos^{-1}\frac{4}{5}\right)$

$\cos^{-1}\frac{5}{13}$ $x=5$ $r=13$ $y=12$

$\sin(\alpha \pm \beta)$

$= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$= \sin\left(\cos^{-1}\frac{5}{13}\right)\cos\left(\cos^{-1}\frac{4}{5}\right) \pm \cos\left(\cos^{-1}\frac{5}{13}\right)\sin\left(\cos^{-1}\frac{4}{5}\right)$

$= \frac{12}{13}\cdot\frac{4}{5} - \frac{5}{13}\cdot\frac{3}{5}$

$= \frac{48}{65} - \frac{15}{65}$

$= \frac{33}{65}$

9. Find the exact value of this expression: $\sin\left(\cos^{-1}\frac{5}{13}-\cos^{-1}\frac{4}{5}\right)$

$\cos 2\theta = 1 - 2\sin^2\theta$

10. Find the exact value of this expression: $\cos\left(2\tan^{-1}\frac{4}{3}\right)$

$\frac{4}{3\theta} = \frac{y}{x}$ $\quad r = 5$

$y = 4$

$x = 3$

$= 1 - 2 \cdot \sin^2(\tan^{-1}\frac{4}{3})$

$= 1 - 2 \cdot \left(\frac{4}{5}\right)^2$

$= 1 - 2 \cdot \frac{16}{25}$

$= 1 - \frac{32}{25}$

$= -\frac{7}{25}$

Extra Credit: solve this equation on the interval $0 \le \theta \le 3\pi$: $\sin(2\theta) = \sqrt{2}\cos\theta$

$2\sin\theta\cos\theta = \sqrt{2}\cos\theta$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + 2k\pi$ $\qquad \frac{3\pi}{4} + 2k\pi$

$\frac{\pi}{4} + 2k\pi$ $\qquad \frac{3\pi}{4} + 2k\pi$

$k = 0$ $\qquad \frac{\pi}{4}$ $\qquad \frac{3\pi}{4}$

$k = 1$ $\qquad \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$ $\qquad \frac{3\pi}{4} + 2\pi = \frac{11\pi}{4}$

~~$k = 2 \qquad \frac{\pi}{4} + 4\pi$~~

$\theta = \frac{\pi}{4}, \frac{9\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{4}$