Math 145 A7 3.3: II 9, III 16, 17 3.4: I 4, II 6, 13 3.5: I 2, II 6, 9, 10
III 20

Note Title 12/16/2005

3.3 II 9 Cantor with 4's and 8's.

Assume that the natural numbers and the real numbers have the same cardinality. Then they can be listed in a one-to-one relationship.

natural numbers	real numbers
1	0.123456.....
2	1.575757.....
3	2.432143.....
4	2.576401.....
5	104.789311.....
⋮	⋮

random assignment

Decide to make a new real number that is different from all the real numbers in the list.

Criteria: ⓐ if the digit chosen in the real number is a 4 call it an 8 in the new real number.

ⓑ if the digit chosen in the real number is not a 4 call it a 4 in the new real number.

ⓒ for real number #1 look at 1st decimal place
for real number #2 look at 2nd decimal place
etc.

new real: 0.44484..... is different from every real number in the list and it is not paired with a natural number

∴ the set of real numbers is larger than the set of natural numbers.

3.3 III 16. This is another Cantor application.

Look back at problem 13 (HW A6) with the red and blue beads. Let the red beads be 1's and the blue beads be 2's. Then the new string of "beads" created by being different from the 1st place of the 1st string, different from the 2nd place of the 2nd string, etc is not in the original pairing to the natural numbers.

∴ The set of all reals made up of only digits 0 and 1 has cardinality greater than the cardinality of the natural numbers.

3.3 III 17 Yes pairs work.
if the pair is not 22 say 46 or 27 when you replace it with 22 at least one of the two digits will be different.
if the pair is 22 and you replace it with 44 both digits will be different.
∴ you will create a new real number, not in the original list.

3.4 I 4. $\{1, 2, 3\}$: list all the subsets

no members	$\{\}$	three of the proper subsets of $\{1,2,3\}$ contain the digit 3.
one member	$\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$	
2 members	$\{1,2\}$ $\{1,3\}$ $\{2,3\}$	
3 members	$\{1,2,3\}$	

3.4 II 6 All in the family of 4 members $\{A, B, C, D\}$
On any one night you could have 0, 1, 2, 3, or 4 family members at the table.

		# of ways
0 members	$\{\}$	1
1 member	$\{A\}$ or $\{B\}$ or $\{C\}$ or $\{D\}$	4
2 members	$\{A,B\}$, $\{A,C\}$, $\{A,D\}$, $\{B,C\}$, $\{B,CD\}$, $\{C,D\}$	6 ways.
3 members	$\{A,B,C\}$, $\{A,B,D\}$, $\{B,C,D\}$, $\{A,C,D\}$	4 ways
4 members	$\{A,B,C,D\}$	1
		16 ways

II 13 Cantor code

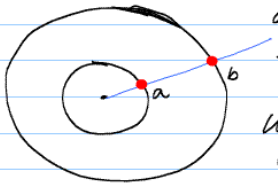
words		missing word?
all	→ {all, infinity, found}	no
you	→ {it, search, them}	yes
infinity	→ {all, them, infinity}	no
found	→ {you, the, it}	yes
them	→ {found, them}	no
search	{all, infinity, search}	no
the	{the, search}	no
it	{infinity, you, all}	yes.

{you, found, it} is a meaningful set.

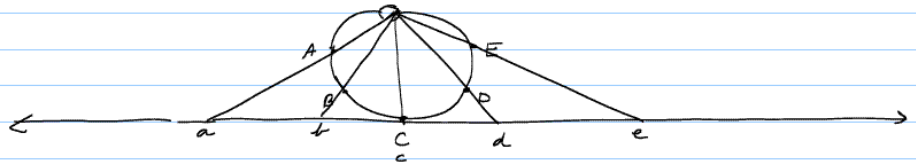
3.5 I 2.

By drawing a line from the vertex to any point on line M you create a one to one correspondence between the points on M and a point on L.

II 6 Place the circles so that they share the same center point. Draw a radius of the inner circle and extend it until it crosses the outer circle. This pairs a point "a" with a point "b". When every radius is drawn, all points on both circles have a pair. ∴ one-to-one correspondence.



3.5 II 10



3.5 III 20. 30 Stereo: Place a sphere, punctured at the north pole on a plane. Pick a point on the sphere. Draw a line through the missing N pole point, the point on the sphere and extend it until the line intersects the plane. There is a one-to-one correspondence between points on the sphere and and points in the plane.