Math 145 A6 3.1: I4, III 16 3.2: I3, II 14, 16, III 30, 32 IV 36
3.3: I 4, II 11, 13, III 19

Note Title 12/9/2005

3.1 I 4. Have each student pick up their backpack.
If every student has one back back and none are left over, it is a one-to-one correspondence.
If one student does not have a pack
or If one student has more than one pack
or If there are left over packs, it is not one-to-one.

3.1 III 16 We know from the last chapter that there are at least two non-bald people with the same number of hairs on their body. Then we could assign each hair of one person to pair with a hair on the other to have a one to one correspondence.
If two sets are the same size they can be put into a 1 to 1 correspondence

3.2 I 3 Set of natural #'s less than 10

$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

set of all even natural #'s

$\{2, 4, 6, 8, \ldots\}$

set of all solutions to the equation $x^2 - 4 = 0$

$\{-2, 2\}$

set of all reciprocals of all natural numbers

$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}$

3.2 II 14

C = set of all cubes of natural numbers

$= \{1, 8, 27, 64, 125, 216, 343, 516, \ldots\}$

$= \{1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3, \ldots, n^3, \ldots\}$

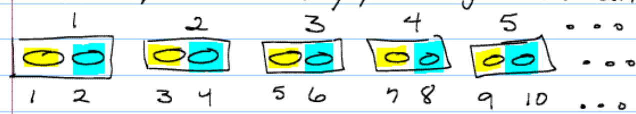
$N = \{1, 2, 3, 4, 5, 6, 7, 8, \ldots, n, \ldots\}$

There is a 1 to 1 correspondence, so the sets have the same cardinality.

3.2 II 16 Merely have each person move into a room of one higher number. Room 1 will be left empty for the new occupant. [Note: There is no largest room number]

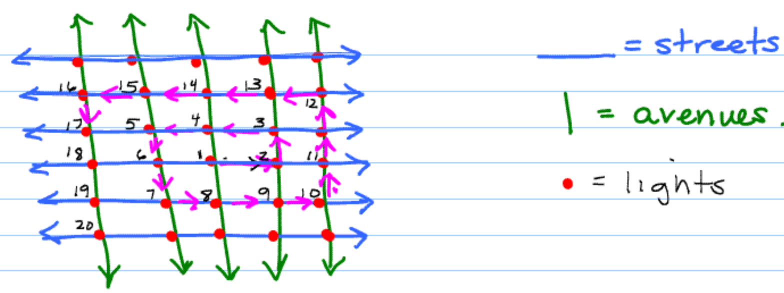
3.2 III 30

Line up the candy packages in an infinitely long line



The individual candies are also countable.
Therefore the set of individual candies has the same cardinality as the set of packages, which is the same cardinality as the natural numbers.

3.2 III 32



Strategy: Pick a light and spiral out from it. You will be able to count all lights and not miss any. There fore there is a 1 to 1 correspondence between the lights and the natural numbers.

3.2 IV 36

in	out
1 – 10	1
11 – 110	2
111 – 1110	3

You can not name one left in the barrel, because all will eventually go out. [even though each time you add more than you take out]

3.3 I 4

3 _ _ _ _

_ 5 _ _ _

_ _ 2 _ _

Think

$\underset{\text{not } 3}{\_\_}\ \underset{\text{not } 5}{\_\_}\ \underset{\text{not } 2}{\_\_}\ \_\_\ \_\_$

A number that does not have what you know about these will give you a number not on this list

One example would pe

4 6 3 2 1

3.3 II 11 Cantor used the 1st decimal of the 1st number, the 2nd decimal digit of the 2nd number, the 3rd decimal digit of the 3rd number, etc.

The visual forms a diagonal

0. X _ _ _ _

0. _ X _ _ _

0. _ _ X _ _

0. _ _ _ X _

3.3 II 13

Suppose the set of all possible colorings is countable and thus has the same cardinality as the natural numbers.

Applying cantors method create a new set whose coloring pattern is:

bead number	is different from
1	set 1 bead 1
2	set 2 bead 2
3	set 3 bead 3
⋮	
n	set n bead n
⋮	⋮

The new set will be different from all of the Counted sets.

∴ The set of all possible colorings of the beads has a cardinality greater than the set of natural numbers.

3.3 III 19. If we could create a group of people who had a 1 to 1 correspondence to the set of real numbers the hotel would not hold them.