math 145 A5 2.3 III 35, 2.6 I 3, II 6, 10; III 30, IV 40, 2.7 I 2; II 7, 10, 20, 23, 25

Note Title 12/8/2005

2.3 III 35 A prime-free gap

Find 6 consecutive natural numbers, none of which are prime

Try a prime grid search.

1	2	3	4	5	6	7	8	9	10
11	12	13	[redacted]	15	16	17	18	19	20
[redacted]	22	23	24	25	26	27	[redacted]	29	30
31	32	33	34	35	36	37	38	39	40
41	[redacted]	43	44	45	46	47	48	49	50
51	52	53	54	55	[redacted]	57	58	59	60
61	62	[redacted]	64	65	66	67	68	69	[redacted]
71	72	73	74	75	76	77	78	79	80
81	82	83	[redacted]	85	86	87	88	89	90
91	92	93	94	95	96	97	[redacted]	99	100

evens are not prime

multiples of 3

multiples of 5

multiples of 7

90, 91, 92, 93, 94, 95, 96 are all not prime.

2.6 I 3

① $\frac{1}{2} + \frac{5}{2} = \frac{6}{2}$

② $\frac{1}{2} - \frac{2}{3} = \frac{3}{6} - \frac{4}{6} = \frac{-1}{6}$

③ $\left(\frac{1}{2}\right) \times \left(\frac{6}{5}\right) = \frac{6}{10}$

④ $\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

⑤ $\frac{\left(\frac{\cancel{5}}{2}\right)\left(\frac{6}{\cancel{5}}\right)}{\left(\frac{2}{3}\right)} = \frac{6}{2} \div \frac{2}{3} = \frac{3\cancel{6}}{1\cancel{2}} \cdot \frac{3}{2} = \frac{9}{2}$

All can be written in form $\frac{a}{b}$ where a and b are integers.
All are rational

2.6 II 6 Given $\sqrt{2}$ is irrational. What about $\frac{3\sqrt{2}}{5\sqrt{2}}$?

$\frac{3\sqrt{2}}{5\sqrt{2}} = \frac{3}{5}$ which is rational.

2.6 II 10. Proof that $\sqrt{5}$ is irrational. is a proof by contradiction.

Assume $\sqrt{5}$ is rational. Then $\sqrt{5}$ can be written as a "reduced" fraction: $\sqrt{5} = \frac{a}{b}$

$$(\sqrt{5})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2 \Rightarrow$$

"a" must be divisible by 5, so $a = 5c \Rightarrow$

$$5b^2 = (5c)^2 \Rightarrow \cancel{5}b^2 = \cancel{5}\cdot 5c^2 \Rightarrow b^2 = 5c^2 \Rightarrow$$

"b" must be divisible by 5 $\Rightarrow \frac{a}{b}$ was not reduced

This is a contradiction.

$\therefore \sqrt{5}$ must be irrational.

2.6 III 30. Show that $\sqrt[3]{2}$ is irrational

Proof by contradiction. (like the one for $\sqrt{2}$ being irrational)

Assume $\sqrt[3]{2}$ is rational, and thus

$\sqrt[3]{2} = \frac{a}{b}$ where a and b have no common factors.

$$(\sqrt[3]{2})^3 = \left(\frac{a}{b}\right)^3 \Rightarrow 2 = \frac{a^3}{b^3} \Rightarrow 2b^3 = a^3 \Rightarrow \text{a has a factor of 2}$$

Since "a" has a factor of 2, we can write $a = 2c$

$\therefore 2b^3 = (2c)^3 \Rightarrow 2b^3 = 2^3c^3 \Rightarrow b^3 = 2^2c^3 \Rightarrow$ b has a factor of 2

Since both "a" and "b" have factors of 2, $\frac{a}{b}$ can be reduced.

This is a contradiction assumption.

$\therefore$ By contradiction $\sqrt[3]{2}$ must be irrational.

2.6 IV 40 Let a and b be any two irrational numbers.

Show that either a+b or a−b must be irrational.

The contradiction of "either a+b or a-b must be irrational" is "a+b and a-b must both be rational."

Let us assume the contradiction is true and see where it leads:

a+b is rational $\Rightarrow$ $a+b = \frac{w}{x}$

a-b is rational $\Rightarrow$ $a-b = \frac{y}{z}$

adding the two lines: $2a = \frac{w}{x} + \frac{y}{z}$

get a common denominator: $2a = \frac{w}{x}\cdot\frac{z}{z} + \frac{y}{z}\cdot\frac{x}{x}$

$2a = \frac{wz + yx}{xz}$

multiply by $\frac{1}{2}$ $a = \frac{wz + yx}{2xz}$ which shows a is rational

This contradicts the assumption. Therefore a+b and a-b cannot both be rational.

2.7 I 2 Moving points

① $10(3.14) = 31.4$

② $1000(0.123123\ldots) = 123.123\ldots$

③ $10(0.4999\ldots) = 4.999\ldots$

④ $\frac{98.6}{100} = .986$

⑤ $\frac{0.333\ldots}{10} = 3.33\ldots$

2.7 II 7: $\sqrt{5}$ is irrational. Irrational numbers cannot be written in the form $\frac{a}{b}$ where a and b are integers.

If $\sqrt{5}$ could be written as a decimal that eventually repeated it could be written in the form $\frac{a}{b}$.

$\therefore$ $\sqrt{5}$ can never be written as a decimal that eventually repeats.

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2.7 II 12: The smallest 10.0398XXXXX is 10.039800000
The largest is 10.039899999.

2.7 II 14: $\frac{6}{7} = 6 \div 7 = 0.857142857142\ldots$

2.7 II 15: $\frac{17}{20} = 17 \div 20 = 0.85$ notice $\frac{17}{20} \cdot \frac{5}{5} = \frac{85}{100}$

2.7 II 24: $x = 71.239999\ldots$

$1000x = 71239.999\ldots$

$100x = 7123.999\ldots$

$900x = 64116$

$x = \frac{64116}{900} = \frac{64116 \div 18}{900 \div 18} = \frac{3562}{50} = \frac{1781}{25}$

note:
① 6+4+1+1+6 = 18 is divisible by 9
② 6 is even ∴ divisible by 2
try reducing by 18

2.7 III 32. No. Once the last 2 was used, the remaining digits would be 1, creating a repeating decimal which is rational.

10, 20, 23, 25

2.7 II 10: 5.7 < the irrational we want < 5.72
Strategy: use an irrational number behind 5.70_ _ _ _ _
one example: 5.7002000200002000000200000002.....
The pattern always has one more 0 digit so it is not repeating.

2.7 II 20. $x = 2.2222\ldots$

$10x = 22.2222\ldots$

$-\ x = 2.2222\ldots$

$9x = 20$

$x = \frac{20}{9}$

2.7. II 25

0.01 001 0001 00001

one zero, two zeros, three zeros, four zeros

The pattern does not repeat ∴ the number is irrational.

2.7 II 23 $x = .0101010101\ldots$

$100x = 01.010101\ldots$

$-\ x = .010101$

$99x = 1$

$x = \frac{1}{99}$