Math 145 A4 Section 2.2 II 22, 24 IV 36, 37 Section 2.3 I 2 II 7, 12, 14, 15, 24

Note Title 12/8/2005

III 32

1, 1, 2, 3, 5, 8, 13, 21, 34, 55

II 22 Start — sticks $50 = 3 + 13 + 34$ — may take any number

you	−3	47	[1, 6]	
friend	−5	42	[1, 10]	42 − 34 = 8
you	−8	34	[1, 16]	
friend	−10	24	[1, 20]	24 − 21 = 3
you	−3	21	[1, 6]	
friend	−6	15	[1, 12]	15 − 13 = 2
you	−2	13	[1, 4]	
friend	−4	9	[1, 8]	9 − 8 = 1
you	−1	8	[1, 2]	
friend	−2	6	[1, 4]	6 − 5 = 1
you	−1	5	[1, 2]	
friend	−2	3	[1, 4]	
you	3	you win		

Always take the smallest fibonacci number

1, 1, 2, 3, 5, 8, 13

2.2

II 24. Answers vary. One possible set of game steps is:

	20			
you	−2	18	[1, 4]	20 − 13 = 7; 7 − 5 = 2
friend	−4	12	[1, 8]	
you	−1	11	[1, 2]	12 − 8 = 4; 4 − 3 = 1
friend	−1	10	[1, 2]	
you	−2	8	[1, 4]	10 − 8 = 2
friend	−4	4	[1, 8]	
you	4	win		

2.2 IV 36. Let F_n = 1st Fibonacci # ; $F_n > 2$

F_{n+1} = next Fibonacci # $= F_n + F_{n-1}$

Can $\frac{F_{n+1}}{F_n} = 2$? $\quad \frac{F_{n+1}}{F_n} = \frac{F_n + F_{n-1}}{F_n} = \frac{F_n}{F_n} + \frac{F_{n-1}}{F_n}$

$$= 1 + \frac{F_{n-1}}{F_n}$$

But $\frac{F_{n-1}}{F_n}$ must be smaller than 1 since $F_{n-1} < F_n$.

So $1 + \frac{F_{n-1}}{F_n} < 1 + 1 = 2$

Therefore $\frac{F_{n+1}}{F_n} < 2$

The only time the ratio is 2 when you have $\frac{2}{1} = \frac{F_{n+1}}{F_n}$

2.2 IV 37. N = natural number, not Fibonacci #

F = largest Fibonacci # smaller than N.

Show that N cannot be written as the sum of two Fibonacci #'s smaller than F.

F must be the sum of the two Fib. #'s before it. $F = F_n + F_{n-1}$. These are the largest Fib. #'s smaller than F and they only add up to F which is smaller than N.

2.3 I 2: $6 = 2 \cdot 3$

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$27 = 3 \cdot 3 \cdot 3$

$35 = 3 \cdot 5$

$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

2.3 II 7. $1 \cdot 2 + 1 = 2 + 1 = 3$ prime
$1 \cdot 2 \cdot 3 + 1 = 6 \cdot 1 = 7$ prime
$1 \cdot 2 \cdot 3 \cdot 4 + 1 = 24 \cdot 1 = 25$ not prime
$\therefore (1 \cdot 2 \cdot 3 \cdots n) + 1$ is not prime (for the first time) when $n = 4$.

2.3 III 32 p is a prime greater than 3
p is an odd number
$p+1$ is an even number
Evens can be divided by two.
$\therefore p+1$ is not prime

Out of order

think
11
11 is odd
$11+1=12$ even
$12/2 = 6$
not prime

2.3 II 12. The key word "only" is missing.
We have not been told if n can be broken down.
All numbers can be divided by themselves $\left(\frac{n}{n} = 1\right)$

All numbers can be divided by 1 $\left(\frac{n}{1} = n\right)$

Even #'s as sums of primes.

2.3 II 14 Goldbach (a solution - there are many)

① $4 = 2+2$	⑥ $14 = 3+11$	⑪ $24 = 5+19$
② $6 = 3+3$	⑦ $16 = 3+13$	⑫ $26 = 7+19$
③ $8 = 3+5$	⑧ $18 = 5+13$	⑬ $28 = 5+23$
④ $10 = 3+7$	⑨ $20 = 3+17$	⑭ $30 = 7+23$
⑤ $12 = 5+7$	⑩ $22 = 3+19$	⑮ $32 = 3+29$

2.3 II Odd Goldbach Odd numbers as sums of two primes
$5 = 2+3$
$7 = 2+5$
$9 = 2+7$
$11 = 2+9$ but 9 is not prime
$11 = 4+7$ but 4 is not prime
$11 = 6+5$ but 6 is not prime
$11 = 8+3$ but 8 is not prime

smallest counterexample is 11.

$\therefore$ not all odd numbers greater than three can be written as a sum of two primes.

2.3 II 24. $n \div 13$ = answer with remainder of 7

$13\overline{)n}$ x R7 $\therefore$ $13x + 7 = 3$ or $\frac{n-7}{13} = x$

If $(n+22) \div 13$, what will the remainder be

$\frac{n+22}{13} = \frac{n}{13} + \frac{22}{13}$ remainder of 9

remainder of 7

combined $9+7=16$

$\frac{16}{13}$ has remainder of 3

OR alternate logic

since $\frac{n-7}{13} = x$ $\quad \frac{n+22}{13} = \frac{n-7+29}{13} = \frac{n-7+26+3}{13}$

$= \frac{n-7}{13} + \frac{26}{13} + \frac{3}{13} = x + 2 + \frac{3}{13}$ remainder.