Math 145 A3 Section 2.2 Mindscapes I 2, II 6, 7, 15, III 29, 30

Note Title 11/5/2005

I 2. The symbol $\varphi$ represents the infinite fractional expression:

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}$$

It is the solution to the equation $\varphi = 1 + \frac{1}{\varphi}$

A sequence of numbers that approach this is the list of the ratios of consecutive Fibonacci numbers.

II 6. Start with 2 baby bunnies (b, b). Bunnies mature (BB) and reproduce in one month and reproduce one month later. No bunnies die.

| | | | |
|---|---|---|---|
| month 0 | $b_1 b_1$ | | |
| month 1 | $B_1 B_1$ | | 1 pair |
| month 2 | $B_1 B_1$, $b_2 b_2$ | | 2 pair |
| month 3 | $B_1 B_1$, $b_3 b_3$ | $B_2 B_2$ | 3 pair = (1+2) pair |
| month 4 | $B_1 B_1$, $b_4 b_4$, $B_3 B_3$ | $B_2 B_2$, $b_5 b_5$ | 5 pair = (2+3) pair |
| month 5 | $B_1 B_1$, $b_6 b_6$, $B_4 B_4$, $B_3 B_3$, $b_8 b_8$ | $B_2 B_2$, $b_7 b_7$, $B_5 B_5$ | 8 pair = (5+3) pair |

Each month the number of pairs is the sum of the pairs the two previous months. $[F_n = F_{n-1} + F_{n-2}]$

II 7. Show a simple formula for calculating $(F_{n+1})^2 + (F_n)^2$

1st list the numbers with their "position:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_n$ | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 | 34 | 55 | 89 ... |
| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 .... |

Look for a pattern.

| | | | | | | |
|---|---|---|---|---|---|---|
| $n$ | 1 | 2 | 3 | 4 | 5 | $n$ |
| $(F_n)^2$ | $1^2$ | $1^2=1$ | $2^2=4$ | $3^2=9$ | $5^2=25$ | $(F_n)^2$ |
| $(F_{n+1})^2$ | $1^2$ | $2^2=4$ | $3^2=9$ | $5^2=25$ | $8^2=64$ | $(F_{n+1})^2$ |
| Sum | 2 | 5 | 13 | 34 | 89 | |
| | $F_3$ | $F_5$ | $F_7$ | $F_9$ | $F_{11}$ | $F_{(2n+1)}$ |

II 15. You cannot create area by rearranging the pieces. If you look very carefully the pieces do not line up perfectly.

III 29.

| number | $\sqrt{F_3/F_1}$ | $\sqrt{F_4/F_2}$ | $\sqrt{F_5/F_3}$ | $\sqrt{F_6/F_4}$ | $\sqrt{F_7/F_5}$ | $\sqrt{F_8/F_6}$ | $\sqrt{F_9/F_7}$ | $\sqrt{F_{10}/F_8}$ |
|---|---|---|---|---|---|---|---|---|
| | $\sqrt{2/1}$ | $\sqrt{3/1}$ | $\sqrt{5/2}$ | $\sqrt{8/3}$ | $\sqrt{13/5}$ | $\sqrt{21/8}$ | $\sqrt{34/13}$ | $\sqrt{55/21}$ |
| | 1.414 | 1.732 | 1.581 | 1.632 | 1.612 | 1.620 | 1.617 | 1.618 |

which is very close to the golden ratio

$\sqrt{F_{n+2}/F_n}$ approaches the Golden Ratio as n gets larger.

III 30. Tribonacci

0, 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136

$\frac{504}{274} = 1.8394$ $\frac{927}{504} = 1.83928$ $\frac{1705}{927} = 1.83926$ $\frac{3136}{1705} = 1.83926$

The ratio seems to be approaching 1.83926