

I 4. 16 new CD's, 5 shelf rack,

Can you place CD's so that there are 3 or fewer on each rack?

Can you place exactly 3 on each shelf?

no & no For each shelf to have three or fewer there must be at most 15 CD's.

II 8. There are 64 squares on a checkerboard. One coin is placed on the first, 2 coins on the second, and each square gets 2 (number on previous square)

$$1 + 2 + 4 + 8 + 16 + 32 + 64 + \dots =$$

$$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + \dots + 2^{n-1} + \dots + 2^{63} = \text{a big number but how big?}$$

Look at this pattern:

$$(1+2) + 4 + 8 + 16 + 32 + 64 + \dots =$$

$$(3 + 4) + 8 + 16 + 32 + 64 + \dots =$$

$$\begin{aligned}
 (7+8) + 16 + 32 + 64 + \dots &= \\
 (15+16) + 32 + 64 + \dots &= \\
 (31+32) + 64 + \dots &= \\
 63 + 64 + \dots &
 \end{aligned}$$

The sum up to a square is always the number on the next square minus one:  $(2^n - 1)$

$\therefore$  The sum of the 64 squares is  $2^{64} - 1$  pieces of gold.  
 which is more than  $1.8 \times 10^{19}$  pieces of gold.

18,000,000,000,000,000,000  
 pieces of gold.

II 15. Start with any natural number ( $n$ )

if  $\left\{ \begin{array}{l} n \text{ is even} \\ n \text{ is odd} \end{array} \right.$  divide by 2

multiply by 3 then add 1.

Repeat until you arrive at 1.

Start with 19

• 19 odd  $19(3) + 1 = 58$

58 even  $58/2 = 29$

Start with 30

30 even  $30/2 = 15$

15 odd  $15(3) + 1 = 46$

29 odd	$29(3) + 1 = 88$	46 even	$46/2 = 23$
88 even	$88/2 = 44$	23 odd	$23(3) + 1 = 70$
44 even	$44/2 = 22$	70 even	$70/2 = 35$
• 22 even	$22/2 = 11$	35 odd	$35(3) + 1 = 106$
• 11 odd	$11(3) + 1 = 34$	106 even	$106/2 = 53$
34 even	$34/2 = 17$	53 odd	$53(3) + 1 = 160$
17 odd	$17(3) + 1 = 52$	160 even	$160/2 = 80$
52 even	$52/2 = 26$	80 even	$80/2 = 40$
26 even	$26/2 = 13$	40 even	$40/2 = 20$
13 odd	$13(3) + 1 = 40$	20 even	$20/2 = 10$
40 even	$40/2 = 20$	10 even	$10/2 = 5$
20 even	$20/2 = 10$	5 odd	
10 even	$10/2 = 5$	16	
5 odd	$5(3) + 1 = 16$	8	
16 even	$16/2 = 8$	4	
8 even	$8/2 = 4$	2	
4 even	$4/2 = 2$	1 end	
2 even	$2/2 = 1$		
1 end			