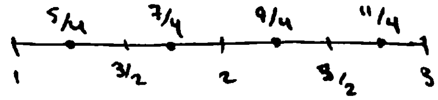
Math 252 Quiz 1 Name Key

Please work the problems below in a neat, organized manner. Show enough detail for me to follow you solution. Thanks.

1. Use a Riemann sum of four subintervals where x_i^* is the midpoint of the ith subinterval to approximate the definite integral $\int_1^3 \frac{1}{x}\,dx$



$\Delta x = \frac{1}{2}$

$$\sum_{i=1}^{4} f(x_i^*)\Delta x = \frac{1}{2}\left(\frac{1}{5/4} + \frac{1}{7/4} + \frac{1}{9/4} + \frac{1}{11/4}\right) = \frac{1}{2}\left(\frac{4}{5} + \frac{4}{7} + \frac{4}{9} + \frac{4}{11}\right)$$

$$= 0.5(2.17951) \approx \boxed{1.09}$$

2. Let $f(x) = \int_{e^x}^{5} \ln(t)\,dt$. Find $f'(x)$.

$$= -\int_5^{e^x} \ln(t)\,dt \longrightarrow f'(x) = -\ln(e^x)\cdot \frac{d}{dx}e^x = \boxed{-xe^x}$$

3. Use the evaluation theorem (FTC II) to find the exact value of $\int_1^2 \frac{2x^2 - 3x + 1}{x^2}\,dx$

$$\int_1^2 2 - \frac{3}{x} + \frac{1}{x^2}\,dx = \int_1^2 2 - \frac{3}{x} + x^{-2}\,dx$$

$$= \left(2x - 3\ln|x| - \frac{1}{x}\right)\Big|_1^2 = 2 - 3(\ln 2 - 0) - \left(\frac{1}{2} - 1\right)$$

$$= \boxed{\frac{5}{2} - 3\ln 2} \approx 0.4206$$

4. Find the family of antiderivatives given by $\int 5\cos x - \sqrt[3]{x}\,dx$.

$$\int 5\cos x - x^{1/3}\,dx = \boxed{5\sin x - \frac{3}{4}x^{4/3} + C}$$

5. A population of flies is increasing at a rate of $2t^2 + 4$ flies per hour over the time interval $0 \le t \le 10$ hours. By how much has the fly population increased between 3 and 7 hours?

$$\int_3^7 2t^2 + 4\,dt = \left(\frac{2}{3}t^3 + 4t\right)\Big|_3^7 = \frac{2}{3}(7^3 - 3^3) + 4(7-3)$$

$$= \frac{2}{3}(343 - 27) + 4(4)$$

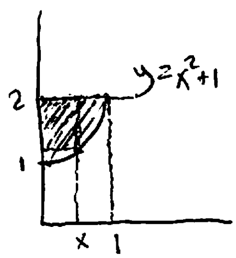
$$= \frac{2}{3}(316) + 16 = \frac{680}{3} \approx \boxed{226.67}$$

Math 252 **Quiz 2** Name: KEY

Please present your work in a clear, neat and organized fassion.

Find the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve $y = f(x) = x^2 + 1$, the y axis and the line $y = 2$ about the x-axis by using:

1. cross-sectional areas.



$$A(x) = \pi\left(2^2 - (x^2+1)^2\right) = \pi(4 - x^4 - 2x^2 - 1)$$

$$= \pi(-x^4 - 2x^2 + 3)$$

$$V = \int_0^1 A(x)\,dx = \boxed{\pi \int_0^1 -x^4 - 2x^2 + 3\,dx}$$

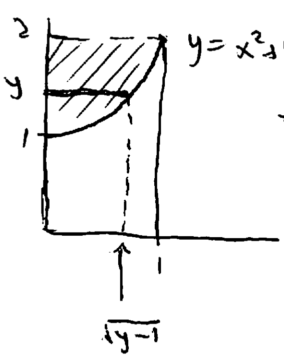
$$-\frac{3}{15} - \frac{10}{15} + \frac{45}{15}$$

$$= \pi\left(-\frac{1}{5}x^5 - \frac{2}{3}x^3 + 3x\right)\Big|_0^1 = \left(-\frac{1}{5} - \frac{2}{3} + 3\right)\pi$$

$$\begin{array}{r} 45 \\ -13 \\ \hline 32 \end{array}$$

$$= \boxed{\frac{32}{15}\pi} \approx 2.13\pi$$

2. cylindrical shells.



$$y = x^2 + 1 \Rightarrow y - 1 = x^2$$

$$\Rightarrow x = \sqrt{y-1}$$

$$\boxed{\int_{y=1}^{2} 2\pi y (y-1)^{1/2}\,dy}$$

$u = y - 1$ $\quad du = dy$

$y = u + 1.$ $\quad y = 1 \Rightarrow u = 0$

$\quad y = 2 \Rightarrow u = 1$

$$V = 2\pi \int_0^1 (u+1)u^{1/2}\,du = 2\pi \int_0^1 u^{3/2} + u^{1/2}\,du$$

$$V = 2\pi\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right)\Big|_0^1 = 2\pi\left(\frac{2}{5} + \frac{2}{3}\right)$$

$$= 2\pi\left(\frac{6}{15} + \frac{10}{15}\right) = 2\pi\left(\frac{16}{15}\right) = \boxed{\frac{32\pi}{15}}$$