Final Exam

Name______________________

Be sure to read the problems carefully and respond directly to the questions asked. Be complete and neat with your solutions and circle your answer.

1. Use the integration techniques discussed in class (substitution, integration by parts, trig substitution, partial fraction decomposition, etc. . .) to find the antiderivatives or exact values of the definite integrals below

   (a) $\int e^t \cosh(3e^t - 2)\, dt$ Hint: substitution

   (b) $\int_0^{\pi} x \sin x\, dx$ Hint: parts

   (c) $\int \frac{dx}{x^2\sqrt{1+x^2}}$ Hint: trig. substitution

   (d) $\int \frac{x}{x^2 - x - 6}\, dx$ Hint: partial fraction decomposition

   (e) $\int_0^{\pi/2} \cos^3 x\, dx$ Hint: trig. integral

2. Determine whether the improper integrals below converge or diverge. If it converges, give its value.

   (a) $\int_0^{\infty} ae^{-bx}\, dx, \quad a, b > 0.$

   (b) $\int_0^1 \frac{1}{\sqrt[n]{x}}\, dx, \quad n > 1.$

3. Let R be the finite region bounded between the graph of $y = x^n$ and the x-axis over the interval $[0, 1]$, $n > 0$.

   (a) Set up and evaluate a definite integral that gives the area of R in terms of n.

   (b) Set up and evaluate a definite integral to find the volume of the solid (in terms of n) obtained by rotating R about the x-axis using disks.

   (c) Set up and evaluate a definite integral to find the volume of the solid (in terms of n) obtained by rotating R about the x-axis using cylindrical shells

4. Suppose a disk of uniform density ρ is spinning about its center, much like the platter in an old-fashioned record player. By setting up and evaluating a definite integral, find the kinetic enerty of the disk in terms of: the total mass M of the disk, the radius R of the disk and the angular velocity ω of the disk. Notes: the velocity of a particle at radius r from the center of rotation is ωr. The kinetic energy of a particle of mass m with velocity v is $\frac{1}{2}mv^2$. Hint: Consider the kinetic energy dE of a thin band of thickness dr and radius r from the center of rotation.

5. Determine whether the series below converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{5 \cdot 2^n}{3^{2n-1}}$$

6. Determine whether each series below converges or diverges and give reasons for your claims.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n}$

7. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}}$$

8. Find the first four terms of the Maclaurin series expansions for each of the following functions. Hint: Use the Maclaurin series of known functions to get the Maclaurin series of these related functions.

   (a) $f(x) = \dfrac{1}{(1-2x)^2}$.

   (b) $f(x) = \int e^{-x^2}\,dx$. Also use the first four non-zero terms of this series to find the approximate value of $\int_0^1 e^{-x^2}\,dx$. How accurate is this approximation? Explain.

9. Find the first four terms of the Taylor series for the function $f(x) = \sqrt[3]{x}$, centered at $x = 8$. Note: You can check your answer by graphing both $\sqrt[3]{x}$ and the four-term Taylor approximation over an interval centered at $x = 8$; they should agree very well with one-another.