Final Exam

Name____________________

Do the problems on the paper provided. Present your work in a neat, organized and clear manner. Please don't crowd your work together. Provide enough detail for me to see how you're solving the problem.

1. Find each of the following limits. For infinite limits use $\pm\infty$ as appropriate. You can use L'Hôpital's rule where it applies.

   (a) $\displaystyle\lim_{x\to 3}\frac{2x-1}{3x+1}$

   (b) $\displaystyle\lim_{x\to 0}\frac{\sin ax}{bx}$, a, b nonzero constants.

   (c) $\displaystyle\lim_{x\to 2^-}\frac{x+2}{x^2-4}$

   (d) $\displaystyle\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$

2. Find the derivative of each function below.

   (a) $f(x) = 3x^2 - 2\sqrt{x} + \frac{5}{x^3}$

   (b) $y = \sin x + \tan(x^3)$

   (c) $g(x) = x^2 e^x$

   (d) $h(x) = \dfrac{\ln x}{x^2}$

3. The equation $2xy + \pi\sin y = 2\pi$ describes a curve in the x-y plane.

   (a) Show that the point $(1, \pi/2)$ is on this curve.

   (b) Use implicit differentiation to find $y'|_{(1,\pi/2)}$.

4. Use logarithmic differentiation to find the derivative of $y = x^{\sin x}$ and then evaluate $y'|_{x=\pi/2}$.

5. A spherical snowball's volume V is related to its radius r by $V = \dfrac{4}{3}\pi r^3$. Suppose that at the moment that $r = 3$cm, the volume is decreasing at a rate of $2\text{cm}^3/\text{sec}$. Implicitly differentiate both sides of the equation with respect to time t to find the rate at which the radius is decreasing when $r = 3$cm.

6. Let $f'(x) = \dfrac{x^2-1}{x^2+1}$ be the derivative of a function $f(x)$.

   (a) Find all critical points of f.

   (b) Use f' to find the intervals where f is increasing, the intervals where f is decreasing and any local extreme values.

   (c) Use f'' to find the intervals where the graph of f is concave up, the intervals where the graph of f is concave down and any inflection points on the graph of f.

   (d) Suppose the point $(2, 1)$ is on the graph of f. Find the equation of the tangent line to the graph of f at this point.

7. Find the most general antiderivative.

   (a) $\displaystyle\int \left(\frac{2}{\sqrt[3]{x}} - 5\sin(3x) \right) \, dx$ (b) $\displaystyle\int \left(3e^{-x} + \frac{1}{x} \right) \, dx$

8. Consider the function $f(x) = -\frac{1}{8}x^3 + 8$ over the interval $[0, 4]$.

   (a) Find $\displaystyle\int f(x) \, dx$.

   (b) Set up and evaluate a Riemann sum where the interval $[0, 4]$ is partitioned into $n = 4$ subintervals and the c_k are the left endpoints of the subintervals.

   (c) Use the evaluation theorem (the second part of the Fundamental Theorem of Calculus) to evaluate $\displaystyle\int_0^4 f(x) \, dx$.

   (d) What is the exact area bounded by the graph of f and the x-axis over the interval $[0, 4]$?

   (e) Note that the number found in part (8b) is larger than that in part (8c). Draw a picture to show why this is so.

9. Do one of the bulleted problems below. You must set up an appropriate function and use the methods of Calculus to answer the question.

   - Find the area of the largest rectangle whose base is on the x-axis and whose upper vertices (corners) are on the graph of $y = -2x^2 + 12$.
   - A hockey team plays in an arena with a seating capacity of $15,000$ spectators. With the ticket price set at $12 per ticket, average attendance at a game has been $11,000$ people. A survey shows that for each $1 that the ticket price is lowered, average attendance will increase by $1,000$ people. How should the owners of the team set the ticket price in order to maximize the total revenue from ticket sales? Hint: Revenue = (price per ticket)(number of tickets sold).

EC The horizontal distance x of an object shot up into the air at an angle of elevation (angle measured up from the horizontal) θ, $0° \leq \theta \leq 90°$ and with a velocity v_0 is given by $x = \frac{v_0^2}{g}\sin(2\theta)$, where g is the acceleration due to gravity. Find the value of θ (in degrees) that maximizes the horizontal distance x.