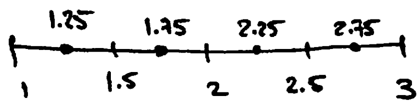
M.252
F'09

Exam 1 Key

1. $\int_1^3 \sqrt{x}\,dx$



$\Delta x = 0.5$

$$\approx \sum_{i=1}^{4} f(x_i^*)\Delta x = 0.5\left(\sqrt{1.25} + \sqrt{1.75} + \sqrt{2.25} + \sqrt{2.75}\right)$$

$$= 0.5(5.599)$$

$$\approx \boxed{2.8}$$

2a. $\int_a^b 3f(x) - 2g(x)\,dx = 3\int_a^b f(x)\,dx - 2\int_a^b g(x)\,dx = 3(-5) - 2(3)$

$$= -15 - 6 = \boxed{-21}$$

b. $\int_b^c f(x)\,dx = \int_b^a f(x)\,dx + \int_a^c f(x)\,dx = -\int_a^b f(x)\,dx + \int_a^c f(x)\,dx$

$$= -(-5) + 2 = \boxed{7}$$

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3. $g(x) = \int_1^x \sin(t^2)\,dt.$ $\quad g'(x) = \frac{d}{dx}\int_1^x \sin(t^2)\,dt = \sin(x^2)$

$$\Longrightarrow g''(x) = \frac{d}{dx}\sin(x^2) = \boxed{2x\cdot\cos(x^2)}$$

4a. $\int_1^3 \sqrt{x}\,dx = \int_1^3 x^{1/2}\,dx = \left(\int x^{1/2}dx\right)\Big|_1^3 = \frac{2}{3}x^{3/2}\Big|_1^3 = \frac{2}{3}(3^{3/2}-1)$

$\approx \frac{2}{3}(4.196) = \boxed{2.797}$

b. $\int_0^{\pi} \cos x \cdot \sin^2 x\,dx$ LET $u = \sin x \Rightarrow du = \cos x\,dx$

$x=0 \to u = \sin 0 = 0$

$x = \pi \to u = \sin \pi = 0$

$\int_0^0 u^2\,du = \boxed{0}$ $\frac{1}{3}u^3\Big|_0^0 = \frac{1}{3}\sin^3 x\Big|_0^0$

5a. $\int \frac{1}{t\sqrt{t}}\,dt = \int \frac{1}{t^{3/2}}\,dt = \int t^{-3/2}\,dt = \boxed{-2t^{-1/2} + C \quad \text{OR} \quad \frac{-2}{\sqrt{t}} + C}$

b. $\int e^x (e^x - 1)^{1/2}\,dx.$ LET $u = e^x - 1 \Rightarrow du = e^x\,dx$

$\int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C$ SO, WE GET $\boxed{\frac{2}{3}(e^x-1)^{3/2} + C}$

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6. $r(t) = T'(t) = -14e^{-0.2t}$ °C/min

$\Delta T = \int_{t=1}^{5} T'(t)\,dt = \int_1^5 -14e^{-0.2t}\,dt$

LET $u = -0.2t \Rightarrow du = -0.2dt$ OR $dt = -\frac{du}{0.2}$

IF $t=1,\ u = -0.2(1) = -0.2$

$t=5,\ u = -0.2(5) = -1$

$\Delta T = \int_{-0.2}^{-1} -14e^u \cdot -\frac{du}{0.2} = \frac{14}{0.2}\int_{-0.2}^{-1} e^u\,du = 70(e^{-1} - e^{-0.2})$

$= 70(-0.4509) = \boxed{-31.56°C}$

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EXAM 2 KEY

1a.

$$\begin{array}{ccc} & x^2 & \sin x \\ -D_x \downarrow & 2x & -\cos x \quad \downarrow \int_x dx \\ -D_x \downarrow & 2 & -\sin x \quad \downarrow \int_x dx \\ -D_x \downarrow & 0 & \cos x \quad \downarrow \int_x dx \end{array}$$

$$\int_0^{\pi} x^2 \sin x \, dx = \left(-x^2\cos x + 2x\sin x + 2\cos x\right)\Big|_0^{\pi}$$

$$= -\pi^2\cos\pi + 2\pi\overset{0}{\cancel{\sin\pi}} + 2\cos\pi$$

$$-(0 + 0 + 2\cos 0)$$

$$= -\pi^2(-1) + 2(-1) - 2(1)$$

$$= \boxed{\pi^2 - 4}$$

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b. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-9}}$

$$\frac{\sin^2\theta + \cos^2\theta = 1}{\cos^2\theta} \longrightarrow \tan^2\theta + 1 = \sec^2\theta$$

$$\text{OR} \quad \sec^2\theta - 1 = \tan^2\theta$$

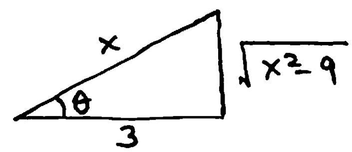
LET $x = 3\sec\theta \longrightarrow dx = 3\sec\theta\tan\theta\, d\theta$

$$\sqrt{x^2-9} = \sqrt{9\sec^2\theta - 9} = 3\sqrt{\sec^2\theta - 1} = 3\tan\theta$$

INTEGRAL BECOMES:

$$\int \frac{\cancel{3}\sec\theta \cdot \cancel{\tan\theta}\, d\theta}{9\sec^2\theta \cdot \cancel{3}\cancel{\tan\theta}} = \frac{1}{9}\int \frac{1}{\sec\theta}\, d\theta = \frac{1}{9}\int \cos\theta\, d\theta = \frac{1}{9}\sin\theta + C$$

NOW, $x = 3\sec\theta \longrightarrow \dfrac{x}{3} = \sec\theta$



SO $\sin\theta = \dfrac{\sqrt{x^2-9}}{x}$

$$\therefore \int \frac{dx}{x^2\sqrt{x^2-9}} = \boxed{\frac{1}{9}\frac{\sqrt{x^2-9}}{x} + C}$$

1c. $\int \frac{x}{x^2+5x+4}\,dx$. $\quad \frac{x}{x^2+5x+4} = \frac{x}{(x+4)(x+1)} = \frac{A}{x+4} + \frac{B}{x+1}$

$\longrightarrow \quad x = A(x+1) + B(x+4)$

IF $x = -1$, $\quad -1 = 3B \Longrightarrow B = \frac{-1}{3}$

IF $x = -4$, $\quad -4 = -3A \longrightarrow A = \frac{4}{3}$

SO $\int \frac{x}{x^2+5x+4}\,dx = \frac{4}{3}\int \frac{dx}{x+4} + -\frac{1}{3}\int \frac{dx}{x+1}$

$$= \boxed{\frac{4}{3}\ln|x+4| - \frac{1}{3}\ln|x+1| + C}$$

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d. $\int \frac{\cos x + \sin 2x}{\sin x}\,dx = \underset{①}{\int \frac{\cos x}{\sin x}\,dx} + \underset{②}{\int \frac{\sin 2x}{\sin x}\,dx}$

① LET $u = \sin x$, $du = \cos x\,dx$

$\int \frac{du}{u} = \ln|u| = \ln|\sin x|$

② $\int \frac{\sin 2x}{\sin x}\,dx = \int \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}}\,dx = \int 2\cos x\,dx = 2\sin x$

SO WE GET : $\boxed{\ln|\sin x| + 2\sin x + C}$

2. $\int_0^2 \ln x \, dx$. $\ln x$ NOT DEFINED AT $x=0$ SO WE TRY

EVALUATING THIS AS: $\lim_{t\to 0^+} \int_t^2 \ln x \, dx$.

$\int \ln x \, dx$ BY PARTS: $u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx \quad v = x$

$$= x\ln x - \int x \cdot \frac{1}{x} \, dx = x\ln x - x$$

$$\lim_{t\to 0^+} \left(x\ln x - x \right)\Big|_t^2 = \lim_{t\to 0^+} \left(2\cdot\ln 2 - 2 - t\cdot\ln t + t \right)$$

FOR $\lim_{t\to 0^+} t\cdot\ln t$ USE L'HÔPITAL'S RULE, WRITING

$t\cdot\ln t$ AS $\frac{\ln t}{(1/t)} = \frac{\ln t}{t^{-1}}$ (INDET FORM $\frac{\infty}{\infty}$ AS $t\to 0^+$)

$$\lim_{t\to 0^+} t\cdot\ln t = \lim_{t\to 0^+} \frac{\ln t}{t^{-1}} \overset{L.H}{=} \lim_{t\to 0^+} \frac{t^{-1}}{-1t^{-2}} = \lim_{t\to 0^+} (-t) = 0$$

$$\therefore \int_0^2 \ln x \, dx = \boxed{2\ln 2 - 2}$$

3. $\Delta x = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$. $\quad S_n = \frac{1}{3}\Delta x \left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + f(x_n) \right)$

$x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$, $x_4 = 3$, $x_5 = 3.5$, $x_6 = 4$

$$S_6 = \frac{1}{3}\cdot\frac{1}{2}\left(\ln 1 + 4\ln 1.5 + 2\ln 2 + 4\ln 2.5 + 2\ln 3 + 4\ln 3.5 + \ln 4 \right)$$

SO $\boxed{S_6 \approx 2.545}$

E.C

$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x} = x^{-1} \longrightarrow f''(x) = -x^{-2}$

$\Rightarrow f^{(3)}(x) = 2x^{-3} \Longrightarrow f^{(4)}(x) = -6x^{-4} = \frac{-6}{x^4}$

SO $|f^{(4)}(x)| = \frac{6}{x^4}$ WHICH DECREASES OVER $[1, 4]$.

THEREFORE $|f^{(4)}(x)|$ IS MAXIMUM AT $x = 1$, WITH A VALUE OF 6.

I.E $|f^{(4)}(x)| \leq 6$ OVER $[1,4]$ SO LET $K = 6$.

$\therefore$ $|E_s| \leq \frac{6(3)^5}{180 \cdot 6^4}$ OR $|E_s| \leq \frac{1}{160}$ OR $\boxed{|E_s| \leq 0.00625}$