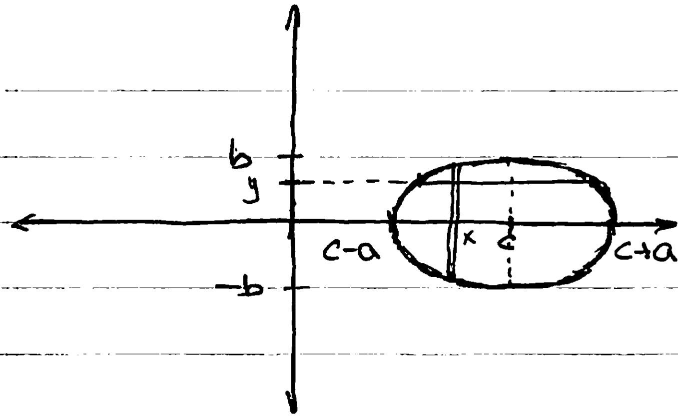
1.




$$\frac{(x-c)^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2(x-c)^2 + a^2y^2 = a^2b^2$$
$$b^2(x-c)^2 = a^2b^2 - a^2y^2$$
$$(x-c)^2 = a^2 - \frac{a^2}{b^2}\cdot y^2$$

$$x-c = \pm\sqrt{\frac{a^2b^2-a^2y^2}{b^2}} = \pm\frac{a}{b}\sqrt{b^2-y^2}$$

$$x = c \pm \frac{a}{b}\sqrt{b^2-y^2}, \quad -b \le y \le b$$

$$dV = \pi\,(x_2^2 - x_1^2)\,dy = \pi\,(x_2-x_1)(x_2+x_1)\,dy$$
$$= \pi\left(\frac{2a}{b}\sqrt{b^2-y^2}\right)(2c)\,dy$$

$$= \frac{4\pi ac}{b}\sqrt{b^2-y^2}\,dy$$

$$V = \int_{-b}^{b} dV = \int_{-b}^{b} \frac{4\pi ac}{b}\sqrt{b^2-y^2}\,dy = \frac{8\pi ac}{b}\int_0^b \sqrt{b^2-y^2}\,dy$$

$$y = b\sin\theta, \quad dy = b\cos\theta\,d\theta, \quad b^2-y^2 = b^2(1-\sin^2\theta) = b^2\cos^2\theta$$

$$V = \frac{8\pi ac}{b}\int_0^{\pi/2} b^2\cos^2\theta\,d\theta = 8\pi acb\int_0^{\pi/2}\frac{1}{2}(1+\cos 2\theta)\,d\theta$$

$$= 4\pi abc\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = 4\pi abc\left[\pi/2 - 0 + 0 - 0\right]$$

$$\boxed{V = 2\pi^2 abc}$$

$$b^2(x-c)^2 + a^2y^2 = a^2b^2 \Rightarrow a^2y^2 = a^2b^2 - b^2(x-c)^2$$

$$\rightarrow y^2 = b^2 - \frac{b^2}{a^2}(x-c)^2 = \frac{b^2a^2 - b^2(x-c)^2}{a^2} = \frac{b^2}{a^2}\left(a^2-(x-c)^2\right)$$

$$\Rightarrow y = \pm\frac{b}{a}\sqrt{a^2-(x-c)^2}$$

$$dV = 2\pi x\left(\frac{2b}{a}\sqrt{a^2-(x-c)^2}\right)dx$$

$$V = \int_{c-a}^{c+a} dV = \frac{4\pi b}{a}\int_{c-a}^{c+a} x\sqrt{a^2-(x-c)^2}\,dx$$

LET $u = x-c \rightarrow x = u+c$, $du = dx$

$x = c-a \Rightarrow u = -a$

$x = c+a \rightarrow u = a$

$$V = \frac{4\pi b}{a}\int_{-a}^{a}(u+c)\sqrt{a^2-u^2}\,du$$

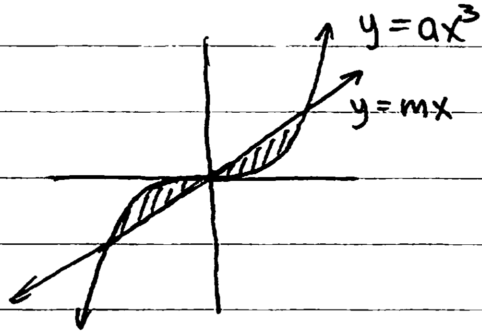
$$= \frac{4\pi b}{a}\left[\cancelto{0}{\int_{-a}^{a} u\sqrt{a^2-u^2}\,du} + c\int_{-a}^{a}\sqrt{a^2-u^2}\,du\right]$$

$$= \frac{4\pi b}{a}\left[2c\int_{0}^{a}\sqrt{a^2-u^2}\,du\right] \qquad u = a\sin\theta,\ du = a\cos\theta\,d\theta$$

$$= \frac{8\pi bc}{a}\int_{0}^{\pi/2} a^2\cos^2\theta\,d\theta = 8\pi abc\int_{0}^{\pi/2}\frac{1}{2}(1+\cos 2\theta)\,d\theta$$

$$= 4\pi abc\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = 4\pi abc\left[\pi/2 + 0 - 0 - 0\right]$$

$$\boxed{V = 2\pi^2 abc}$$




$$ax^3 = mx$$

$$\longrightarrow \quad ax^3 - mx = 0$$

$$\longrightarrow \quad x(ax^2 - m) = 0$$

$$\longrightarrow \quad x = 0, \quad x = \pm\sqrt{\frac{m}{a}}$$

$$c = 2 \cdot \int_0^{\sqrt{m/a}} (mx - ax^3)\,dx = 2\left(\frac{1}{2}mx^2 - \frac{1}{4}ax^4\right)\Big|_0^{\sqrt{m/a}}$$

$$\Longrightarrow \quad c = m\left(\frac{m}{a}\right) - \frac{1}{2}a\left(\frac{m^2}{a^2}\right)$$

$$\Longrightarrow \quad c = \frac{m^2}{a} - \frac{1}{2}\cdot\frac{m^2}{a} \quad \text{or} \quad c = \frac{1}{2}\frac{m^2}{a} \quad \text{or} \quad \boxed{m^2 = 2ac}$$