

Be sure to quickly look through the exam to get an idea of its length and the type of problems involved. First do the problems with which you have confidence, then attack other less obvious ones. Be neat and show only the work you want me to see. Please circle your final solutions. No credit is given for solutions only.

1. Evaluate each expression below for  $x = -2, y = 3, z = -5$ .

(a)  $x - 2y + z^2$

$$= -2 - 2(3) + (-5)^2$$

$$= -2 - 6 + 25 \rightarrow -8 + 25 = \boxed{17}$$

(b)  $x - 2(y + z)^2$

$$-2 - 2(3 + (-5))^2 = -2 - 2(-2)^2 = -2 - 2(4) = -2 - 8 = \boxed{-10}$$

2. Perform the indicated operations and simplify each expression. Combine like terms where possible, reduce any fractions to lowest terms, and express your solution using positive exponents only.

(a)  $3(x^2 - 4x + 2) - x(3x + 5)$

$$= \cancel{3x^2} - 12x + 6 - \cancel{3x^2} - 5x = \boxed{-17x + 6}$$

(b)  $(2x^2)^3 \cdot (3x)^{-2}$

$$8x^6 \cdot \frac{1}{9}x^{-2} = \boxed{\frac{8}{9}x^4}$$

(c)  $\frac{(2wy^{-3})^4}{6y^{-3}}$

$$= \frac{16w^4y^{-12}}{6y^{-3}} = \frac{8}{3}w^4y^{-9} = \boxed{\frac{8w^4}{3y^9}}$$

(d)  $(2x - 3)^2$

$$= \boxed{4x^2 - 12x + 9}$$

(e)  $(a + b)(a^2 - ab + b^2)$

$$= a^3 - \cancel{a^2b} + \cancel{ab^2} + \cancel{a^2b} - \cancel{ab^2} + b^3 = \boxed{a^3 + b^3}$$

(f)  $\frac{m^2 - m - 6}{m^2 + m} \div \frac{m^2 - 4m + 3}{m^2 - 1}$

$$= \frac{(m-3)(m+2)}{m(m+1)} \cdot \frac{(m+1)(m-1)}{(m-3)(m-1)} = \boxed{\frac{m+2}{m}}$$

(g)  $\frac{5}{x^2 - 3x} - \frac{x - 4}{x^2 - 8x + 15}$

$$\frac{(x-5) \cdot 5}{(x-5)x(x-3)} - \frac{x(x-4)}{x(x-3)(x-5)} = \frac{5x - 25 - x^2 + 4x}{x(x-3)(x-5)}$$

$$= \frac{-1(x^2 - 9x + 25)}{x(x-3)(x-5)}$$

(h)  $\frac{\left(1 + \frac{2}{x} + \frac{1}{x^2}\right)x^2}{\left(1 - \frac{1}{x^2}\right)x^2}$

$$\frac{x^2 + 2x + 1}{x^2 - 1} = \frac{(x+1)(x+1)}{(x+1)(x-1)} = \frac{x+1}{x-1}$$

3. Simplify the radical expression  $5\sqrt{7} - 2\sqrt{28} + 6\sqrt{63}$  and provide a decimal approximation.

$$= 5\sqrt{7} - 2 \cdot 2\sqrt{7} + 6 \cdot 3\sqrt{7}$$

$$= 5\sqrt{7} - 4\sqrt{7} + 18\sqrt{7} = 19\sqrt{7} \approx 50.27$$

4. Completely factor each polynomial below.

(a)  $3x^3y^4 - 12x^2y^3 + 15x^4y$

$$3x^2y(xy^3 - 4y^2 + 5x^2)$$

(b)  $9m^2 - 25$

$$(3m-5)(3m+5)$$

(c)  $x^2 + x - 20$

$$(x+5)(x-4)$$

	5t	11	
2t	10t <sup>2</sup>	22t	11 \cdot 2 \cdot 5
-1	-5t	-11	

$-110t^2$

(d)  $10t^2 + 17t - 11$

$$(2t-1)(5t+11)$$

(e)  $2a - 6b - ax + 3bx$

$$2(a-3b) - x(a-3b)$$

$$(a-3b)(2-x)$$

(f)  $4x^2 - 20x + 25$

$$(2x-5)^2$$

$\frac{36}{36}$

5. Solve each equation or inequality below. If it's an inequality, graph the solution set on a number line. Give exact answers in simplified form (reduce fractions, simplify radicals).

(a)  $*7 - 3(2x - 5) = 4x - 6$

$\Rightarrow 7 - 6x + 15 = 4x - 6$

$\Rightarrow 22 - 6x = 4x - 6$

$\Rightarrow 28 = 10x \Rightarrow x = \frac{28}{10} = \frac{14}{5}$

$x = \frac{14}{5}$

30. (b)  $\left(\frac{2x}{3} - \frac{x}{5}\right) = \left(\frac{3x}{2} + 1\right) - 30$

$\Rightarrow 20x - 6x = 45x + 30$

$\Rightarrow 14x = 45x + 30$

$\Rightarrow -31x = 30 \Rightarrow x = -\frac{30}{31}$

$x = -\frac{30}{31}$

(c)  $(\sqrt{x-5})^2 = (x-7)^2$

$\Rightarrow x-5 = x^2 - 14x + 49$

$\Rightarrow 0 = x^2 - 15x + 54$

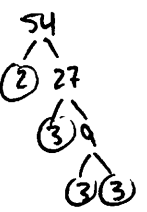
$\Rightarrow 0 = (x-9)(x-6)$

$\Rightarrow x=9$  or  $x=6$

$\sqrt{9-5} = 9-7$   
 $\sqrt{4} = 2 \checkmark$

$\sqrt{6-5} = 6-7$   
 $\sqrt{1} = -1$  NO

$x=9$



(d)  $(2x-5)^2 = 24$

$\Rightarrow 2x-5 = \pm\sqrt{24} = \pm 2\sqrt{6}$

$\Rightarrow 2x = 5 \pm 2\sqrt{6} \Rightarrow x = \frac{5 \pm 2\sqrt{6}}{2}$

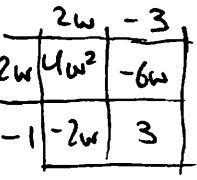
$x = \frac{5 \pm 2\sqrt{6}}{2}$

(e)  $*4w^2 = 8w - 3$

$\Rightarrow 4w^2 - 8w + 3 = 0$

$\Rightarrow (2w-3)(2w-1) = 0 \Rightarrow$

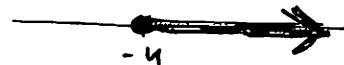
$w = \frac{3}{2}$  or  $w = \frac{1}{2}$



(f)  $-4 \leq \frac{2y-4}{3}$

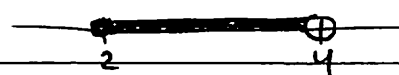
$\Rightarrow -12 \leq 2y-4$

$\Rightarrow -8 \leq 2y \Rightarrow -4 \leq y$  or  $y \geq -4$



(g)  $-3 < 5 - 2x \leq 1$

$\Rightarrow -8 < -2x \leq -4$



Dec. 17, 2008  $\Rightarrow 4 > x \geq 2$

$\Rightarrow 2 \leq x < 4$

$\frac{28}{28}$

6. Solve the formula  $z = \frac{2x - 3y}{4}$  for  $y$ .

$$\Rightarrow 4z = 2x - 3y$$

$$\Rightarrow \frac{4z - 2x}{-3} = y$$

$$\Rightarrow 4z - 2x = -3y$$

$$\Leftrightarrow \boxed{\frac{2x - 4z}{3} = y}$$

7. Given the quadratic equation  $2m^2 - 4m - 3 = 0$ , do the following:

(a) Solve the equation by the method of completing the square.

$$m^2 - 2m - \frac{3}{2} = 0$$

$$m^2 - 2m = \frac{3}{2}$$

$$m^2 - 2m + 1 = \frac{5}{2}$$

$$\Rightarrow (m-1)^2 = \frac{5}{2}$$

$$\Rightarrow m-1 = \pm \sqrt{\frac{5}{2}}$$

$$\Rightarrow \boxed{m = 1 \pm \sqrt{\frac{5}{2}}}$$

(b) \*Solve the equation using the quadratic formula. Do not give calculator approximations; leave the answer in simplified radical form.

$$m = \frac{4 \pm \sqrt{16 - 4 \cdot 2(-3)}}{4} = \frac{4 \pm \sqrt{16 + 24}}{4} = \frac{4 \pm \sqrt{40}}{4}$$

$$= \frac{4 \pm 2\sqrt{10}}{4} = \frac{2(2 \pm \sqrt{10})}{4} = \boxed{\frac{2 \pm \sqrt{10}}{2}}$$

8. \*Answer the question by solving an equation or system of equations. A 2000 seat theater has tickets for sale at \$15 and \$22. How many tickets should be sold at each price for a sellout performance to generate a total revenue of \$33,500?

$x = \#15 \text{ TICK.}$

$y = \#22 \text{ TICK}$

1500 OF THE \$15 TICK.

AND 500 OF THE \$22 TICK

WERE SOLD

$$x + y = 2000 \Rightarrow y = 2000 - x$$

$$15x + 22y = 33,500$$

$$15x + 22(2000 - x) = 33,500$$

$$15x + 44,000 - 22x = 33,500$$

$$-7x = -10,500 \Rightarrow x = 1500 \text{ AND } y = 500$$

9. \*Find an equation of the line that passes through the points  $(4, -1)$  and  $(-6, 5)$ .

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{-6 - 4} = \frac{6}{-10} = -\frac{3}{5}$$

$$y = -\frac{3}{5}x + \frac{7}{5}$$

$$y = mx + b \quad \text{with } (4, -1) \rightarrow -1 = -\frac{3}{5}(4) + b$$

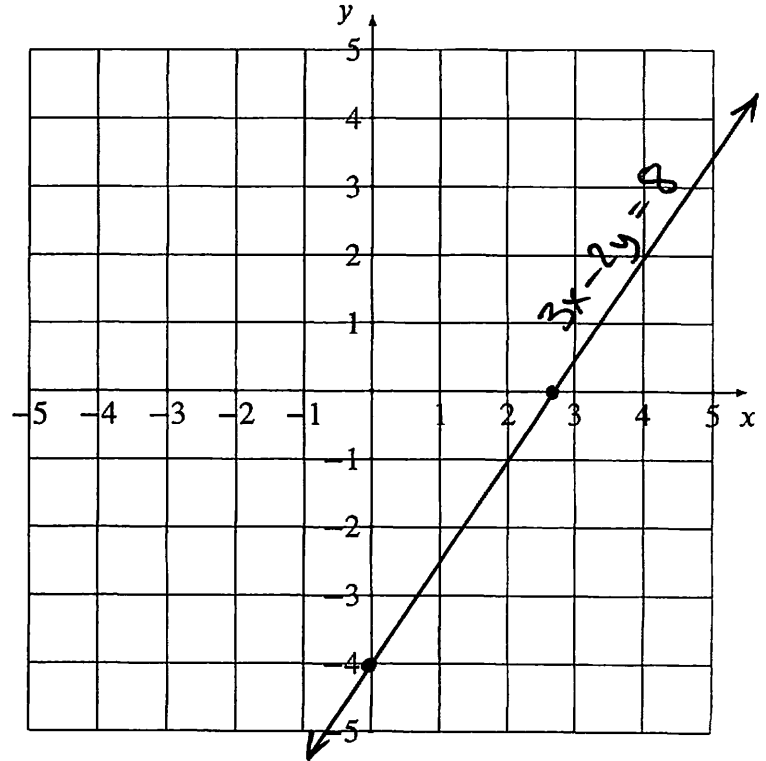
$$\rightarrow -1 = -\frac{12}{5} + b \rightarrow b = -\frac{4}{5} + \frac{12}{5} = \frac{8}{5}$$

10. \*Graph the line  $3x - 2y = 8$  in the coordinate system to the right.

x	y
$2\frac{2}{3}$	0
0	-4

$$3x = 8 \rightarrow x = \frac{8}{3} = 2\frac{2}{3}$$

$$-2y = 8 \rightarrow y = -4$$



11. \*Solve the system of equations below. You must use an algebraic method of solution (either elimination or substitution.)

$$\begin{array}{r} 3x - 4y = 18 \\ -2(5x - 2y = 16) \end{array} \rightarrow \begin{array}{r} 3x - 4y = 18 \\ -10x + 4y = -32 \end{array}$$

$$-7x = -14 \rightarrow x = 2$$

$$3(2) - 4y = 18$$

$$\rightarrow 6 - 4y = 18$$

$$\rightarrow -4y = 12$$

$$\rightarrow y = -3$$

Solution is  $(2, -3)$