Some *Conservation of Momentum* Examples

These example problems are straight from Page 9 of *Conceptual Physics, 10th ed.*, by Paul G. Hewitt.

Example 1

A car crashes into a wall at 25 m/s and is brought to rest in 0.1 s. Calculate the average force exerted on a 75 kg test dummy by the seat belt.

Solution to Example 1

We assume that the seat belt does all of the stopping of the test dummy– in other words, we ignore any friction from the seat, normal (support) forces from air bags etc.

We focus our attention on the dummy and forget the car. In terms of its initial velocity ($v_\circ = 25\ m/s$) and its original mass ($m_\circ = 75\ kg$), the dummy's original momentum, $P_\circ$[1], is given by:

$$P_\circ = m_\circ \times v_\circ$$

$$P_\circ = (75\ kg) \times (25\ m/s)$$
$$P_\circ = 1,875\ (kg \cdot m)/s \quad ^2$$

The dummy's momentum after the collision, P_f is determined by its final mass ($m_f = mm_\circ$) and its final velocity ($v_f = m$):

$$P_f = m_f \times v_f$$

$$P_f = (75\ kg) \times (0\ m/s) = 0$$

We immediately see that the dummy's momentum changed by $-1,875(\ kg \cdot m)/s^3$.

When an object interacts with something else, it experiences an *impulse* (I), which is defined as the product of the (average) force acting on that object and the time that force acts. It is also equal how much the object's momentum changes ΔP during that interaction:

$$I = force \times time = \Delta P$$

In this case, we know $\Delta P = -1,875(\ kg \cdot m)/s$, we know the time of interaction is 0.1 s, so we can use this relation to find the average force acting on the dummy:

$$force = \frac{\Delta P}{time} = \frac{-1,875(\ kg \cdot m)/s}{0.1\ s}$$
$$force = -18,750\ N$$

The minus sign just means the force is pointing in the opposite direction as the original momentum. Since we know that momentum points in the same direction as the velocity, that means that the force points in the opposite direction as the original velocity– the case where things slow down. This, of course, makes sense!

As a side note, let's just calculate the dummy's average acceleration. We know that the dummy's velocity changed by 25 m/s (Δv), in a time of 0.1 s. Therefore we can use:

$$\Delta v = acceleration \times time$$

$$acceleration = \frac{\Delta V}{time}$$
$$acceleration = \frac{25\ m/s}{0.1\ s}$$
$$acceleration = 250\ m/s^2 \approx 25\ g$$

So this dummy was undergoing 25 g's That's quite a lot!

Example 2

Judy (mass 40.0 kg), standing on slippery ice, catches her leaping dog, Atti (mass 15 kg), moving horizontally at 3.0 m/s. hat is the speed of Judy and her dog after the catch?

Solution to Example 2

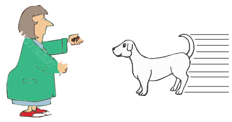
A diagram of what is going on is shown below in Figure .

Since Judy and the dog are moving together as

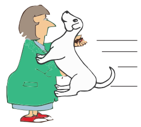
[1] P is the most common variable to use for momentum in physics. Yup, it seems kinda weird... but I guess m was already taken!

[2] Note that the symbol $\cdot$ means 'multiply,' just like $\times$. I'm using it just to separate the kg unit from the m unit

[3] It started with $1,875(\ kg \cdot m)/s$, ended with zero, so the seat belt changed its momentum by $-1,875(\ kg \cdot m)/s$



(a) Before Judy catches her dog



(b) After Judy catches her dog

Figure 1: Judy catches her dog

one unit (they're 'stuck' together) after the collision, this is an example of a perfectly inelastic collision.

The original momentum of the **Judy+Dog** system, $\vec{P}_\circ$ is given by:

$$\vec{P}_\circ = \vec{P}_{\circ,J} + \vec{P}_{\circ,D}$$

Where $\vec{P}_{\circ,J}$ is Judy's original momentum and $\vec{P}_{\circ,D}$ is the dog's original momentum. Since Judy is originally not moving, $\vec{P}_{\circ,J} = 0$. We calculate the dog's original momentum in the usual manner:

$$\vec{P}_{\circ,D} = m_D \times \vec{v}_{\circ,D}$$

$$\vec{P}_{\circ,D} = (15\ kg) \times (3.0\ m/s\ \ Leftward)$$

$$\vec{P}_{\circ,D} = 45\ (kg \cdot m)/s\ \ Leftward$$

So that for the entire Judy+Dog system:

$$\boxed{\vec{P}_\circ = 45\ (kg \cdot m)/s\ \ Leftward}$$

Momentum conservation tells us that the total momentum of a (closed) system is constant– so that the momentum of the **Judy+Dog** system is the same before Judy catches the dog as afterward.

For the momentum after the catch, $\vec{P}_f$, we write:

$$\vec{P}_f = (m_J + m_D) \times \vec{v}_f$$

$$\vec{P}_f = (40\ kg + 15\ kg) \times \vec{v}_f$$

$$\vec{P}_f = (55\ kg) \times \vec{v}_f$$

Note that we used the sum $(m_J + m_D)$ for the mass since Judy and the dog are 'stuck' together. For the same reason, we only need to discuss one velocity, $\vec{v}_f$. This is typical of inelastic collisions.

Using momentum conservation:

$$\vec{P}_f = \vec{P}_\circ$$

$$(55\ kg) \times \vec{v}_f = 45\ (kg \cdot m)/s\ \ Leftward$$

$$\vec{v}_f = \frac{45\ (kg \cdot m)/s}{55\ kg}\ \ Leftward$$

$$\boxed{\vec{v}_f = 0.818\ m/s\ \ Leftward}$$

Example 3

Comic-strip hero Superman meets an asteroid in outer space and hurls it at 800 m/s, as fast as a bullet. The asteroid is a thousand times more massive than Superman. In the strip, Superman is seen at rest after the throw. Taking physics into account, what would be his recoil velocity?

Solution to Example 3

This is another conservation of momentum problem, of course. It differs from the others in that the actual masses of Superman and the asteroid aren't specified. But, as we'll see, this doesn't matter. First, lets define a few things:

m_s=Mass of Superman
m_a=Mass of asteroid
$\vec{v}_{\circ,s}$=Superman's original velocity
$\vec{v}_{\circ,a}$=The asteroid's original velocity
$\vec{v}_{f,s}$=Superman's final velocity
$\vec{v}_{f,a}$=The asteroid's final velocity

Here's what we know from the problem's set-up:

m_s=$1,000\ m_a$
$\vec{v}_{\circ,s}$=$0\ m/s)$
$\vec{v}_{\circ,a}$=$0\ m/s)$
$\vec{v}_{f,a}$=$800\ m/s\ \ Forward$

We are, of course, looking for $\vec{v}_{f,s}$. I just chose the direction $Forward$ for the asteroid's final velocity arbitrarily. In general, you'll find it enormously helpful to make little tables like these when you're solving problems using conservation of momentum (and later, conservation of energy, conservation of angular momentum, etc.)

It may seem that since we don't know the masses of Superman and the asteroid, this is a hopeless problem. But we'll see that knowing their relative masses is more than adequate!

Since this is a momentum conservation problem, the first thing to do is figure out the original momentum of the **Superman+asteroid system**, aka $\vec{P}_\circ$. I won't bother writing out equations for this because both the asteroid and Superman have initial velocities of 0 m/s, so that the total momentum of the system is zero:

$$\boxed{\vec{P}_\circ = 0}$$

The final momentum of the system, $\vec{P}_f$ is a bit more confusing. Here it is:

$$\vec{P}_f = (m_a \times \vec{v}_{f,a}) + (m_s \times \vec{v}_{f,s})$$

$$\vec{P}_f = (1,000 m_s \times 800\ m/s\ \ Forward) + (m_s \times \vec{v}_{f,s})$$

In the second equation, I just used the fact that $m_a = 1,000\ m_s$. Now we know that $\vec{P}_f = \vec{P}_\circ = 0$:

$$((1,000 m_s) \times (800\ m/s\ \ Forward)) + (m_s \times \vec{v}_{f,s}) = 0$$

$$((800,000\ m_s)\ m/s\ \ Forward) + (m_s \times \vec{v}_{f,s}) = 0$$

Now we see why only the *relative* masses matter! If we talk about everything in terms of a Superman Mass, we see that the actual mass of Superman is just a scale factor! And scaling zero doesn't matter. In case this seems confusing, look at the math:

$$(m_s \times \vec{v}_{f,s}) = -(800,000\ m_s)\ m/s\ \ Forward$$

Divide by m_s on each side of the equation:

$$\vec{v}_{f,s} = -800,000\ m/s\ \ Forward$$

$$\boxed{\vec{v}_{f,s} = 800,000\ m/s\ \ Backward}$$

Note that I just used the fact that moving with a negative velocity in the *Forward* direction is the same as moving with a positive velocity in the *Backward* direction.

Bottom line: Superman is really moving fast after he throws that asteroid!

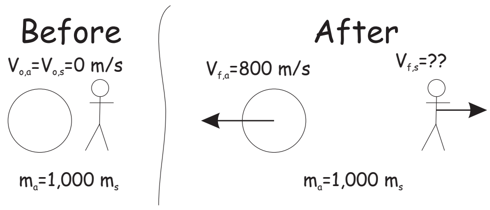
Some General Comments

I hope this helps you in solving conservation of momentum problems, or at least thinking about them. The methods used to solve conservation of momentum problems translate to a bunch of different things, in this class and out.

Here's a general method for approaching these problems:

1 Make a table of all of the information the problem gives you, as was done in the Superman/asteroid problem. This may also include subtle things like whether or not a collision is inelastic (in which case there is only one final velocity of the system you need to worry about) or elastic.

2 Draw a sketch if it helps you think about what is going on. Personally, I like to draw sketches for everything. I like to draw 'before' and 'after' sketches and list all the variables under them to help me keep the variables and numbers straight in my mind. If I were solving the Superman problem, I'd do something like this:



In the momentum conservation problems, I also find that making a table like the one below can help. Here I'm using the variables from the superman example:

Initial Momenta	*Final Momenta*
$\vec{P}_{\circ,s} = 0$	$\vec{P}_{f,s} = m_s \times \vec{v}_{f,s}$
$\vec{P}_{\circ,a} = 0$	$\vec{P}_{a,s} = 800,000\, m_s\ (m/s\ For.)$

3 Determine what you need to know– what are you looking for? A velocity, force, time?

4 The hard part: choose a method which takes all of this information and helps you solve the problem. Determine whether or not you use momentum conservation, etc.