Physics 100: **Solutions** to Homework Assignment #6

Was Due on Monday, March 12^{th} at the Beginning of Class

Section 1. **Warm-up! Fill-in-the-Blanks (1 pt each)**

1. To calculate the work done by a force on something, you multiply the component of the ___force___ in the direction of motion, by the ___distance___ over which the force acted.
2. ___Power___ is the rate at which energy is expended, and is calculated by dividing the amount of ___work___ done by the ___time___ it is done.
3. The energy due to the position of something or the movement of something is known as ___mechanical energy___
4. The mechanical energy of something due to its position is ___potential energy___. The mechanical energy of something due to its motion is ___kinetic energy___.
5. Energy cannot be created or destroyed, but it may be transformed from one form into another. In other words, energy is ___conserved___.

Section 2. **Short Answer Questions (2 pts. each)**

6. One kilogram of butter has roughly 11 times as much energy in it as one kilogram of TNT, a well-known explosive. Why, then, are you allowed to bring butter on an airplane, while TNT is banned?

   Answer: The rate at which the energy can be extracted from butter is so low that it will never cause an explosion. TNT, though lower in total chemical energy per gram, gives up all of its energy all at once, leading to explosions. In other words, TNT is capable of providing more power that butter, even though it doesn't provide as much energy. It is the power that matters during an explosion.

7. One gram of TNT, a well-known explosive, has an energy of rough 2,700 J. **(A)** How fast would a one gram rock (a pebble) have to have in order to have the same energy? **(B)** How much energy does an asteroid with a mass of 500 kg have if it is traveling at 30 km/s, relative to the Earth?

   Answer: (A) The kinetic energy of a pebble is given by:

$$KE_{pebble} = \frac{1}{2} mass \times speed^2$$

   For this to equal the same amount of chemical energy in one gram of TNT, we have the equation:

$$\frac{1}{2} mass \times speed^2 = 2,700\ J$$

   We must remember to convert all of our units into kg, m, s so that we'll have the units of Joules in our expression for kinetic energy:

$$\frac{1}{2}(0.001\ kg) \times speed^2 = 2,700\ J$$

$$speed^2 = \frac{2 \times 2,700\ J}{0.001\ kg} = 5,400,000\ m^2/s^2$$

$$speed = \sqrt{5,400,000\ m^2/s^2} = 2,324\ m/s$$

That's pretty darn fast!
(B) An asteroid that has a mass of 500 kg and moving at 30 km/s has a kinetic energy of[1]:

$$KE = \frac{1}{2} mass \times speed^2$$

$$KE = \frac{1}{2}(500\ kg) \times (30,000\ m/s)^2$$

$$KE = 2.25 \times 10^{11}\ J$$

That's a pretty big number! This is just for a 500 kg asteroid moving at 30 km/s. The asteroid that hit the earth 65 million years ago and killed all the dinosaurs had a speed of about 30,000 km/s=30,000,000 m/s, but was roughly the *size of Mt. Everest*, so that it had a kinetic energy around 100,000,000,000 (10^11) *tons* of TNT, aka, 100 *tetratons.* This energy is equivalent to more than 10,000 times the energy stored in the combined nuclear arsenals of the U.S. and Soviet Union at their peak. For more information, see *Nemesis*, a book by Richard Muller.

8. Give an example of a case in which a force is exerted on an object without doing work on the object.

   Answer: An example might be the the weight of a book, sitting at rest on a table. Since it is at rest, the book is not moving, so no work is being done. ($Work = Force \times Displacement$– if the $Displacement$ is zero (not moving), then the $work$ is zero)

9. Two cars are raised to the same elevation on service-station lifts. If one car is twice as massive as the other, how do their potential energies compare?

   Answer: Gravitational potential energy is equal to:

   $$PE_{grav} = mass \times (accel.\ due\ to\ gravity) \times height$$

   The acceleration due to gravity is the same for both cars, and they are at the same height. The only difference is that one has twice the mass as the other. Therefore the more massive one will have twice the (gravitational) potential energy.

10. A 2,000 kg car is moving at 20 m/s and brakes to a stop in order to avoid hitting a cute, cuddly little kitten in the street. **(A)** How much work did the car's brakes do on the car in order to stop it? **(B)** If the car stopped in 30 m distance, what was the force of friction acting on the tires?

    Answer: (A) Before breaking, the car had a kinetic energy of:

    $$KE_{before} = \frac{1}{2}(2,000\ kg) \times (20\ m/s)^2$$

    $$KE_{before} = 400,000\ J$$

    After breaking, the car is not moving, so it has a kinetic energy of zero. The *work-energy* theorem tells us that an the amount of work done on an object by an external force is equivalent to the amount by which that object's kinetic energy changes. In this case, we see that friction must have done -400,000 J of work, since that is how much the car's kinetic energy changed.

    (B) Using the definition of work:

    $$Work = force \times displacement$$

    We have:

    $$-400,000\ J = Force_{friction} \times (30\ m)$$

    $$Force_{friction} = -13,333\ N$$

    The negative sign just means that friction is acting in a direction opposite to the velocity– thus slowing down the car (taking away kinetic energy).

[1] Remember: to calculate things correctly, we have to use the proper units: kilograms, meters, and seconds

11. You push a crate, starting from rest, across a factory floor for 10 m. The crate has a mass of 50 kg. You push with a force of 100 N, and friction resists with a force of 70 N. **(A)** List four forces acting on the crate. **(B)** What is the net force on the crate? **(C)** How much work is done on the crate after 10 m? **(D)** How much work did you do against friction? **(E)** How much work did you do total? **(F)** How fast is the crate moving after 10 m of your pushing?

Answer: **(A)** The four forces acting on the crate are our old friends:

- Gravity (Weight)
- Support Force (Normal Force)
- Friction
- Force you're pushing with (also a support or normal force)

(B) The net force is equal to 30 N, in the direction you're pushing (the direction the crate is moving). The Support Force of the floor cancels out the crates weight, and friction acts in the opposite direction you push in, so it acts to cancel out your efforts.
(C) The net force on the crate is 30 N, in the direction the crate is moving, so using the definition of work:

$$Work = Force \times distance$$

$$Work = 30\ N \times 10\ m = 300\ J$$

(D) You are using 70 N to cancel friction. So the work you do against friction is:

$$Work_{against\ friction} = 70\ N \times 10\ m = 700\ J$$

(E) You are exerting a total of 100 N, in the direction of motion. So in total, the amount of work you're doing is:

$$Work_{total,\ you} = 100\ N \times 10\ m = 1,000\ J$$

Comparing **(D)** and **(E)**, we see that the majority of energy you're expending on the crate is going into fighting friction– and ultimately heating the floor and making sound.
(F) The net amount of work done on the crate is equal to 300 J, so the work-energy theorem tells us that that is how much the kinetic energy of the crate changes. If it started with zero kinetic energy (it was at rest to begin with), then the amount by which it's KE changed is equal to its final KE. Using the definition of KE:

$$KE = 300\ J = \frac{1}{2} \times mass \times speed^2$$

We solve for the speed:

$$speed = \sqrt{\frac{2 \times 300\ J}{50\ kg}} = 3.5\ m/s$$

12. What will be the kinetic energy of a pine cone that falls from a tree (starting from rest), and undergoes a decrease in potential energy of 30 J?

Answer: As the pine cone falls, its potential energy is converted into kinetic energy, ignoring air resistance, etc. Therefore, if it losses 30 J of potential energy, it has gained 30 J of kinetic energy.

13. As we discussed a few weeks ago, a skydiver jumping out of an airplane will accelerate up a terminal velocity and stop accelerating– in other words the skydiver will fall through the air at constant speed. **(A)** What is keeping the skydiver from accelerating? **(B)** As the skydiver falls at a constant speed, does her kinetic energy remain constant, increase, or decrease? **(C)** As the skydiver falls at constant speed, does her potential energy remain constant, increase, or decrease? **(D)** Reconcile your answers to **(B)** and **(C)**, keeping in mind that energy is conserved. *Hint where does the energy go?*

Answer: **(A)** Air resistance is keeping the sky diver from accelerating.
(B) Because kinetic energy is defined as

$$KE = \frac{1}{2} mass \times speed^2$$

if she falls at constant speed (and isn't changing mass), then her kinetic energy must remain constant.
(C) However, since her gravitational potential energy is defined as:

$$PE_{grav} = mass \times (accel.\ due\ to\ grav.) \times height$$

we immediately see that she is losing potential energy as her height is decreasing.
(D) Energy is conserved– in the universe as a whole, energy is neither created or destroyed, but can be moved around. Energy in a particular system may not be conserved, if the objects that define that system interact with things outside of that system (just like momentum conservation). In this case, the energy of the skydiver does not remain constant because she is interacting with the air. The amount of energy she loses is equivalent to the amount of energy the air gains (in heat, noise, etc.)– so that if we define our system to be the skydiver *and* the air, then the total energy of our system will not change.

14. **(A)** Can momenta cancel? **(B)** Can kinetic energies cancel? If a moving object doubles its speed, by how much does its **(C)** momentum and **(D)** kinetic energy change?

Answer: **(A)** Yes, absolutely! If you have questions about this, come and see me ASAP!
(B) No– kinetic energies cannot cancel. Kinetic energies are positive scalars: they are proportional to mass and the square of speed, they are always positive. Therefore, adding two kinetic energies amounts to adding two positive numbers, which never add to zero (unless the numbers are zero to start with, but this is not really a case of one canceling the other).
(C) Since momentum is proportional to $mass \times velocity$, doubling the speed amounts to doubling the magnitude of the velocity (they are the same thing). Therefore, doubling the speed doubles the magnitude of momentum, too.
(D) Kinetic energy, on the other hand, is proportional to $speed^2$, so if you double the speed, you increase the kinetic energy by a factor of four:

$$KE_{orig} = \frac{1}{2} m \times v^2$$

$$KE_{after} = \frac{1}{2} m \times (2v)^2 = \frac{1}{2} m \times 4(v)^2 = 4KE_{orig}$$

15. What is the efficiency of a machine that miraculously converts all the input energy to useful output energy?

Answer: That machine would be 100% efficient. Let me know if you find one. I have a bridge to sell you... ⌣̈

16. What is the ultimate (single) source of energies for the burning of fossil fuels, dams, and windmills?

Answer: The Sun. The Sun is responsible for evaporating the water that makes the clouds that causes the rain that builds up the water behind the dams. The Sun is the source of food for the plants (and hence the animals who ate the plants) that became fossil fuels (e.g. oil and coal) all those millions of years ago. The Sun is responsible for heating different parts of the Earth, causing pressure gradients and ultimately wind to power the windmills.

17. Which requires more work to stop– a light truck or a heavy truck if they are both moving with the same momentum?

Answer: For the light truck (mass m) and the heavy truck (mass $\mathbf{M}$) to have the same momentum, the light truck must be moving much faster (speed $\mathbf{V}$) than the heavy truck (speed v):

$$momentum = p = m\mathbf{V} = \mathbf{M}v$$

Now, kinetic energy is defined as

$$KE = \frac{1}{2} mass \times speed^2$$

We can write this as:

$$KE = \frac{1}{2}(mass \times speed) \times speed$$

$$KE = \frac{1}{2} momentum \times speed$$

Where it is understood that we are talking about the *magnitude* of the momentum since we are using speed. We are ignoring the direction component of momentum, but this is OK since kinetic energy is not a vector.

Now, since both the trucks have the same momentum, we can see that the light truck has the higher kinetic energy since it has a bigger speed:

$$KE_{light} = \frac{1}{2} p \times \mathbf{V}$$

$$KE_{heavy} = \frac{1}{2} p \times v$$

$$KE_{heavy} < KE_{light}$$

18. Two identical twins (e.g. same mass, etc.) climb the stairs of a building, up to the top. Twin Alpha accomplishes this feat in 30 s and the other, Twin Beta, takes 40 s, **(A)** which twin does more work? **(B)** Which twin uses more power?

    Answer: (A) The twins do the same amount of work, all told. They each raise their (identical) masses the same height, imparting themselves with the same gravitational potential energy.
    (B) Power is defined by the amount of work done per unit time. Since Twin Alpha exerted the same amount of work as Twin Beta but in less time, Twin Alpha used more power.

19. You have a choice of catching either a bowling ball or a baseball, both with the same kinetic energy. Which is safer for you to catch?

    Answer: This is similar to Problem 17. Let's say the bowling ball has a mass of $\mathbf{M}$, and the baseball has a mass of m[2]. For them to have the same kinetic energies amounts to the bowling ball (speed=v) moving slower than the baseball (speed=$\mathbf{V}$):

$$KE = \frac{1}{2}\mathbf{M}v^2 = \frac{1}{2}m\mathbf{V}^2$$

As in Problem 17, we can see that kinetic energy can be written in terms of the momenta of the bowling ball (P_{bowl}) and baseball (P_{base}:

$$KE = \frac{1}{2}P_{bowl}v^2 = \frac{1}{2}P_{base}\mathbf{V}^2$$

Since $P_{bowl}v^2 = P_{base}\mathbf{V}^2$, we see that

$$P_{bowl} > P_{base}$$

[2] $\mathbf{M} \gg m$

Now recall that impulse is the change of momentum that something undergoes. Impulse is also equal to the force something experiences multiplied by the time it experiences the force. If we try to catch the bowling ball and baseball in the same way– taking the same amount of time– we can use this definition of impulse to compare the relative stopping forces required:

$$Impulse = \Delta Momentum = Force \times time$$

In the case of the bowling ball, it has to undergo a larger change in momentum than the baseball does in order to stop. Therefore, a larger stopping force is required, making it more dangerous to catch.

20. If you drop a rubber ball from rest from a height of 3 m, what is the maximum height it will bounce back *and why?*

Answer: It could never bounce higher than it started from because to go higher would require would mean it would have a higher potential energy than when it started. But if there is no force doing work on it to add energy to it, it will never gain extra energy. In reality, the ball won't bounce as high as it started from because there is energy lost to air resistance and deformation of the ball during the time it bounces (leading to heat and noise). A super ball that bounced back to its original height would make no noise or get warm.

21. If an automobile were to have a 100% efficient engine, transferring all of the fuel's energy to work, **(A)** would the engine be warm to your touch? **(B)** Would its exhaust heat the surrounding air? **(C)** Would it make any noise? **(D)** Would it vibrate?

Answer: **(A)** No: there would be no energy lost to heat the engine up if all of the energy in the gasoline was converted to useful mechanical energy.
(B) No: likewise, there would be no energy lost to heat the exhaust and hence the surrounding air.
(C) No: again no energy lost to make noise.
(D) No: no energy to deform the metal of the car (vibrations).

22. How many kilometers per liter will a car obtain if its engine is 25% efficient and it encounters an average retarding force of 500 N at highway speed? Assume that the energy of gasoline is 40,000,000 J/liter.

Answer: The 500 N force required to keep the car moving at constant speed comes from the engine, which burns the gasoline. It is required to counteract the forces of friction and air resistance which would otherwise slow the car down. **The net force on the car is zero since it is moving at constant velocity– a net force is not required to keep something moving**[3].
The engine, after moving 1 km will therefore do this amount of work:

$$Work = (500\ N) \times (1,000\ m) = 500,000\ J$$

If the engine burns one liter of gasoline, only 25% of it is converted to useful work. In other words, there are 10,000,000 J/liter of useful energy in the gasoline. So the amount of gasoline we need to burn to do $500,000\ J$ of work is given by:

$$Amount\ of\ gas = \frac{500,000\ J}{10,000,000\ J/liter} = \frac{1}{20}\ liter$$

So it takes 1/20 of a liter of gas to provide the energy required to move the car 1 km. This is the same as saying that you can get 20 km of travel out of 1 liter of gasoline. This is equivalent to 47 mpg, which is petty good[4].

[3]Please take not of this: this is the property of inertia. A lot of you still seem to think that a net force is needed to keep something moving at a constant velocity. It is not. That is what Aristotle said, and he was wrong. A net force causes an acceleration, which changes the velocity.

[4]1 km=0.62 miles; 1 liter-0.264 gallons; 1 km/liter= 1 0.62 miles)/(0.264 gallons)=2.35 miles/gallon.