Physics 100: **Solutions** to Homework Assignment #5

Was Due on Monday, March 5^{th} at the Beginning of Class

Section 1. **Warm-up! Fill-in-the-Blanks (1 pt each)**

1. In an *elastic collision*, momentum is conserved *(conserved/ not conserved)*.
2. In an *inelastic collision*, momentum is conserved *(conserved/ not conserved)*.
3. To calculate momentum of an object, you multiply its mass by its velocity.
4. To calculate the impulse exerted on something, you multiply the force applied on the object by the time it is applied.
5. Impulse is equal to the change in momentum of an object that the impulse acts on.
6. A collision in which colliding objects rebound without lasting deformation or the generation of heat is a/an elastic collision, while a collision in which colliding objects become distorted, generate heat, and possibly stick together is a/an inelastic collision.
7. The impulse resulting from a force of 12 N acting on a cart with a mass of 1.5 kg for 2.5 s is equal to 30 N s.
8. The momentum of a 4 kg Adélie penguin sliding across an iceberg at 2 m/s is 8 (kg m)/s.
9. The speed of a 4 kg Adélie penguin with a momentum of 3 (kg m)/s is 0.75 m/s.
10. A head-on collision between two cars would be less *(more/ less)* damaging to the occupants if they stuck together than if they rebounded upon impact.

Section 2. **Short Answer Questions (2 pts. each)**

11. What does it mean to say that momentum (or any quantity) is *conserved*?

    Answer: It means that momentum (or the quantity in question) does not change. For example, the total momentum before a collision is the same as after the collision.

12. Which undergoes the greatest change in momentum: (1) a baseball moving at 20 m/s that is caught, (2) a baseball that is thrown at 20 m/s, or (3) a baseball moving at 20 m/s that is caught and then thrown back at 20 m/s?

    Answer: (3) The baseball that is caught at 20 m/s and thrown back at 20 m/s undergoes the greatest change in momentum. As the mass of the ball in all three cases is the same, we need only consider the change in velocity[1]. Let's call the direction the ball is traveling in for (1) positive. Then, in (3), the ball changes its velocity from +20 m/s to -20 m/s, or $\delta v = -40\ m/s$. In (1), the velocity changes from +20 m/s to 0 m/s, or $\delta v = -20\ m/s$. In (2), the ball's velocity changes from 0 to -20 m/s, or $\delta v = -20\ m/s$. We see that the change in momentum in (3) is twice that as in (1) and (2), which actually undergo the same change in momentum.

    Shown below is a graphical representation of what is going on. Each row corresponds to one of the three cases. The left column depicts the state of the baseball before being caught/thrown/caught+thrown. The middle column represents the change in the baseball's momentum during its being (1) caught, (2) thrown, and (3) caught and then thrown. The third column depicts the final momenta of the ball. The arrows all represent momentum, or a change in momentum in the case of the middle column.

[1] Remember, momentum is defined as *mass* × *velocity*.

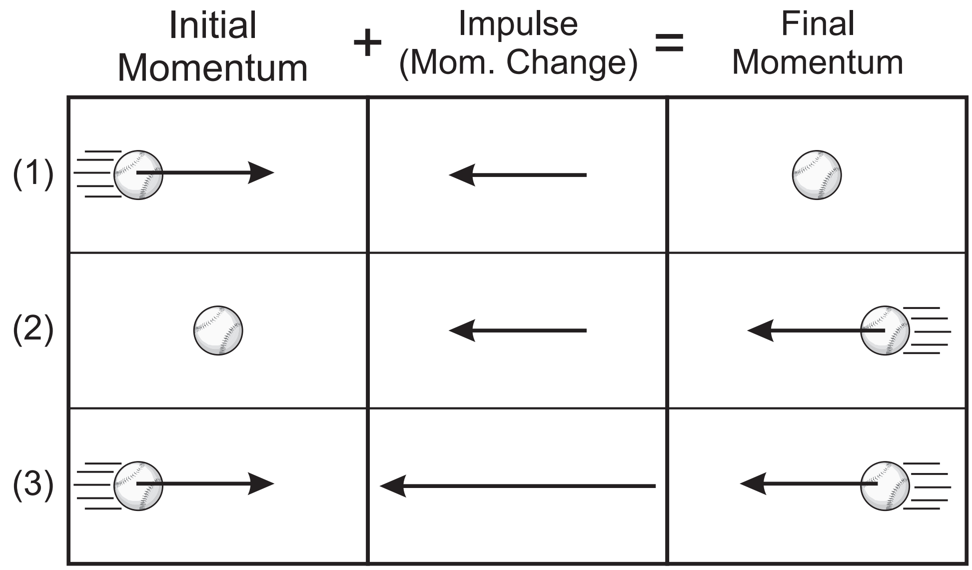


Figure 1: The three different cases of Problem 12

In all three cases, the change in momentum of the ball is determined by realizing that the *final momentum* of the ball is equal to the (vector) sum of its *original momentum* and the *change in its momentum*[2] that it experiences during the cathch/throw/catch+throw[3]. In equation form[4]:

$$\overrightarrow{momentum}_{original} + \overrightarrow{\Delta momentum} = \overrightarrow{momentum}_{final}$$

From this it is obvious that in (1), the change in momentum is equal, but opposite, to its original momentum. In (2), the change in momentum is equal, but opposite, to the original momentum of the ball in (1), in order to get it moving in the opposite direction but the same speed starting from from rest. Case (3) is also a bit more clear. It is combination of the first two cases: the ball is caught the same as in (1) and subsequently thrown as in (2), so gets twice the change in momentum.

13. Why do gymnasts use floor mats that are very thick and spongy?

Answer: When you're landing on the floor and come to a stop, you have to change your momentum. This can be accomplished by using (1) a large force over a short time or using (2) a small force over a long time[5] (in either case, the impulse, or change in the gymnast's momentum, is the same).

The former method (1) puts a lot of force on your body and leads to things like broken bones, etc. This is known as a hard landing! Landing on concrete is an example of a hard landing.

The second method(2) gently slows your body down, leading to fewer broken bones, etc. This is known as a soft landing. Landing on thick, spongy mats is an example of a soft landing.

So the reason gymnasts use thick, spongy floor mats is to reduce the amount of bodily stress (e.g. broken bones) resulting from their activities by reducing the force of the landing by prolonging its duration.

[2] Also known as **impulse**!

[3] Think of it this way: the ball starts with some momentum, the baseball player adds (vectorially) some momentum to it, and the result is the ball's final momentum.

[4] Remember: the arrows over the words remind us that we're talking about vectors!

[5] Of course, you could use a medium force over a medium time, etc., but for the purposes of illustrating the point, we'll only consider the extreme cases

14. When rock climbing, climbers tie themselves to ropes in case they fall. Sometimes climbers can fall quite a ways before the rope catches them. The ropes climbers use are not like those you would buy at the hardware store; they are elastic, kind of like stiff bungee cords. Old ropes can become worn out and there is always a danger that they will break (bad news for the falling climber!). Given the possibility of ropes breaking, explain why climbers prefer to use these special, elastic ropes rather steel cable, which is much stronger and would never break. *Hint: weight is not the issue.*

    Answer: This is very similar to Problem 13. When a rope catches a falling climber, it is best for the climber if the rope slows her down slowly. That is why climbers use stretchy ropes. If they used steel cables, they would be slowed down very quickly, resulting in a lot of force– and that would really hurt.

15. **(A)** When an apple falls from a tree and strikes the ground without bouncing, what becomes of its momentum? **(B)** Is momentum conserved in this case?

    Answer: **(A)** When an apple falls from a tree and strikes the ground without bouncing, it has undergone a perfectly inelastic collision with the Earth. Because momentum is conserved (see the answer to **(B)**), we know that the apple's pre-impact momentum was simply transferred to the apple/Earth body. Because the apple/Earth combination is so massive compared to the apple alone, we do not perceive any final motion of the apple/Earth combination. In equation form:

$$momentum_{initial} = mass_{apple} \times velocity_{apple}$$

$$momentum_{final} = (mass_{apple} + mass_{Earth}) \times velocity_{apple\&Earth}$$

    Conservation of momentum tells us that:

$$momentum_{initial} = momentum_{final}$$

$$mass_{apple} \times velocity_{apple} = (mass_{apple} + mass_{Earth}) \times velocity_{apple\&Earth}$$

$$velocity_{apple\&Earth} = \frac{mass_{apple} \times velocity_{apple}}{mass_{apple} + mass_{Earth}} = \frac{something\ small}{something\ HUGE}$$

$$velocity_{apple\&Earth} = something\ really,\ really\ small^{6}$$

    (B) Momentum is *always* conserved.

16. Shown in Figure 16 is a before and after snapshot of Godzilla jumping straight down onto a (horizontally) moving cart. Before Godzilla lands on the cart, it is moving quickly (Figure 16A). After Godzilla lands in the cart, it (and Godzilla) are moving much slower (Figure 16B). Give **two** reasons for this: (A) One in terms of a horizontal force acting on the cart and (B) one in terms of momentum conservation.

[6] When you divide a small number by a large number, you get something really small! Try it!

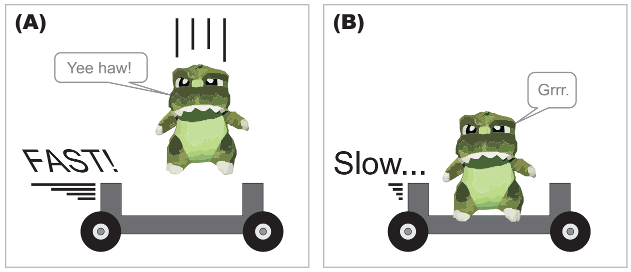


Figure 2: Question 16: (A) The cart is moving quickly just before Godzilla jumps onto it. (B) After Godzilla lands in the cart, it moves much more slowly.

Answer: (A) When Godzilla comes into contact with the cart, the force of friction between the cart and Godzilla acts to accelerate Godzilla so that he is moving in the same direction (and eventually at the same speed) as the cart. Newton's Third Law tells us that the force of friction that accelerates Godzilla to the right is matched with a force on the cart, of equal magnitude but opposite direction (i.e. to the left). Since the cart was originally moving to the right, this means that the cart slows down (its speed decreases).

(B) In terms of momentum conservation, *we only need to consider the horizontal components of momentum.* From Problem 15, we know where the vertical momentum goes (it's conserved in the inelastic Godzilla/cart/Earth vertical collision). So, considering the horizontal components of the momentum (and velocity) only, we use momentum conservation[7]:

$$momentum_{before,horiz} = (mass_{cart} \times velocity_{cart}) + (mass_{Godzilla} \times velocity_{Godzilla})$$

This simplifies a bit, because the initial *horizontal* velocity of Godzilla is zero:

$$momentum_{before} = mass_{cart} \times velocity_{cart}$$

Since the collision between Godzilla and the cart is a perfectly inelastic one, we can write for the final momentum:

$$momentum_{after} = (mass_{cart} + mass_{Godzilla}) \times velocity_{cart\&Godzilla}$$

Now, as always, momentum is conserved, so $momentum_{before} = momentum_{after}$:

$$mass_{cart} \times velocity_{cart} = mass_{cart} + mass_{Godzilla}) \times velocity_{cart\&Godzilla}$$

Solving for $velocity_{cart\&Godzilla}$, we find:

$$velocity_{cart\&Godzilla} = \frac{mass_{cart}}{mass_{cart} + mass_{Godzilla}} \times velocity_{cart}$$

Since $mass_{cart}/(mass_{cart} + mass_{Godzilla}) < 1$, we see that $velocity_{cart\&Godzilla} < velocity_{cart}$. In other words, the cart slows down when Godzilla jumps on it.

Another way to think about this is that since momentum is equal to $mass \times velocity$ and is conserved (always the same), Godzilla's jumping on the cart acts like a sudden increase in mass. So to keep the same momentum, the velocity must drop. Thinking about it in this way leads to the same equation, above.

[7]Note, since we're working in one dimension– the horizontal dimension– vector addition reduces down to simple scalar addition.

17. Jan, Tam, and Sue are three astronauts that are outside a spaceship. All the astronauts weigh the same when they're on the Moon and are equally strong. They decide it would be fun to play a game of astronaut catch. Jan throws Tam to Sue and the game begins! This is shown in Figure 2. The idea is that Sue will catch Tam and throw her back to Jan, at which point the whole sequence will repeat. How long will this game last (i.e. how many times will Tam be thrown)?

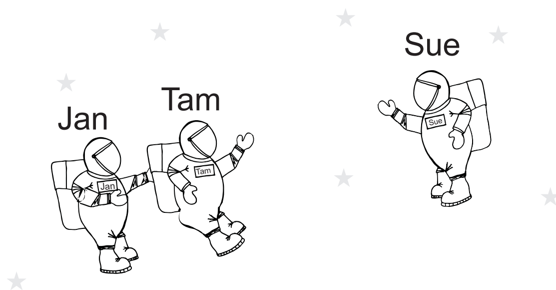


Figure 3: Question 17: Three astronauts play catch.

Answer: Since the astronauts are all equally strong, we can assume this to mean that **each 'toss' represents the same amount of impulse exerted on the astronauts engaged in the toss**. We'll define this impulse as $\vec{P}_\circ$. Since they all have the same mass, that means that this impulse causes the same change in their velocities per toss, which we'll call $\vec{V}_\circ$.

Figure 3 shows a representation of what happens. The different spheres represent the astronauts: $J \leftrightarrow Jan$, $T \leftrightarrow Tam$, and $S \leftrightarrow Sue$.

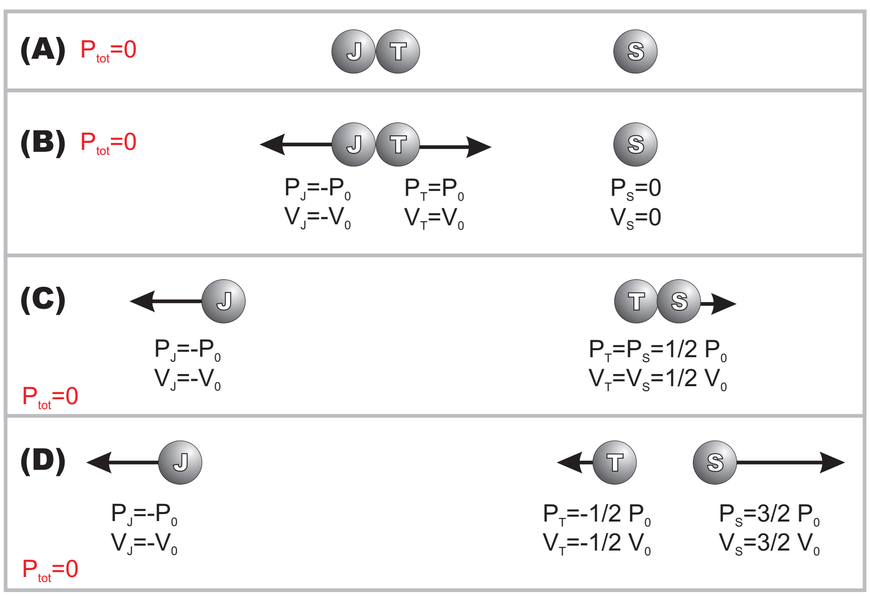


Figure 4: Question 17: The three astronauts represented by spheres.

Figure 3a, depicts the beginning of the game. Nobody is moving, so the net momentum of the system is zero. This is important.

Figure 3b shows what happens just after Jan tosses Tam towards Sue. The toss imparts equal and opposite momentum on Tam and Sue (Newton's Third Law; conservation of momentum), changing Jan and Tam's velocities by $\pm\vec{V}_\circ$[8].

Figure 3c shows what happens in the resulting catch of Tam by Sue (a collision!). If we analyze Tam and Sue as a separate system, we see that

$$momentum_{before} = \vec{P}_\circ$$

Since momentum is always conserved, and the collision is perfectly inelastic, we know that

$$momentum_{after} = \vec{P}_\circ$$

Since they are stuck together, we also know that Tam and Sue have the same final velocity. Since they also have the same mass, they must therefore have the same momentum. This chain of reasoning leads us to this conclusion:

$$momentum_{Tam} = momentum_{Sue} = \frac{1}{2}\vec{P}_\circ$$

Figure 3d shows the situation just after the second toss. Here, Sue and Tam's momenta changed by $\pm\vec{P}_\circ$, just as in the first case. Since Sue had $+1/2\ \vec{P}_\circ$ just before the toss, she ends up with $+3/2\ \vec{P}_\circ$, corresponding to a velocity of $+3/2\ \vec{V}_\circ$. Similarly, Sue had $+1/2\ \vec{P}_\circ$ before the toss, but since her change in momentum was in the negative direction, she ends up with a momentum of $-1/2\ \vec{P}_\circ$, corresponding to a velocity of $-1/2\ \vec{V}_\circ$. We can see a few things now. The total momentum of the system is still zero, as momentum conservation tells us it must. But we also see that Jan is moving to the left twice as fast as Tam. This means that Tam will never catch Jan, so Jan will never be able to toss Tam back to Sue. The game is over after only two tosses.

18. When you are traveling in your car at highway speed, an insect splatters on your windshield, changing its momentum (and surface area! :)) in the process. Compared to the change in momentum of the insect, by how much does the momentum of your car change?

    Answer: It changes by the same amount, but in the opposite direction in order to conserve momentum: if the insect changed its momentum by some amount, say $\Delta\vec{P}$, then to have the initial momentum equal the final momentum, your car must have changed its momentum by $-\Delta\vec{P}$ so that the total change in momentum of the car+insect system is $\Delta\vec{P} + (-\Delta\vec{P}) = 0$.

19. A brick is slides across a table and comes to a stop due to (dynamic) friction. Solve for the force of dynamic friction if the brick has a mass of 2 kg, is initially moving at 3 m/s, and comes to a halt in 4 sec. Ignore air resistance.

    Answer: The initial momentum of the brick is given by the product of its velocity and mass:

$$momentum = mass \times velocity$$

$$momentum = (2\ kg) \times (3\ m/s)$$

$$momentum = 6\ (kg\ m)/s$$

In sliding to a halt, it changes its momentum from 6 (kg m)/s to 0 (kg m)/s in 4 sec. In this case, our change in momentum, aka $\Delta momentum$, is -6 (kg m)/s (the minus sign means it decreases its momentum). Impulse is defined as a change in momentum, which is also equal to the force acting on something times the time it acts:

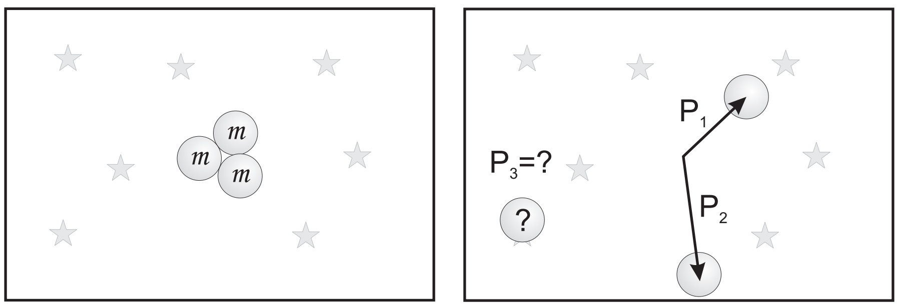
$$impulse = \Delta momentum = force \times time$$

[8]Remember, if two vectors, $\vec{A}$ and $\vec{B}$ have the same magnitude, but opposite directions, we write that as $\vec{A} = -\vec{B}$. So in this case, we are saying that Jan and Tam go off at the same speed but opposite directions

$$-6\ (kg\ m)/s = force \times (4\ s)$$
$$force = \frac{-6}{4}\ (kg\ m)/s^2$$
$$force = -1.5\ N$$

So the force slowing down the brick (dynamic friction) is equal to -1.5 N. The negative sign just means the force is pointing in the opposite direction as the initial momentum (velocity) of the brick, which makes perfect sense.

20. Three identical space probes are stuck together and are sitting at rest as shown in Figure 3(a). Then a small explosive charge sets them moving away from one another, as shown in Figure 3(b). The momenta of the first and second space probes are shown as vectors P_1 and P_2, respectively. Graphically determine what the momentum vector of the third one (P_3) is in this case. Do not calculate anything– draw on Figure 3(b) what the length and direction of the P_3 vector should be, showing how you derived it. Separately, explain your reasoning in words. Note that the location of the third space probe is not correct. **Use Figure 5(a) if you would like a larger diagram to draw on.**



(a) The probes together (b) The probes separate

Figure 5: Question 20: Three probes initially at rest (a) and after a small explosive charge sends them moving away from one another (b).

Answer: Because all the space probes are at rest before they break apart, the initial total momentum of all three balls is equal to zero. Conservation of momentum tells us that the final total momentum must also be zero. Since we know the momenta of two of the three space probes, all we need to do is find the third momentum vector which adds to the first two to make a total momentum of zero. We do this in the usual manner by adding the first two head-to-tail, as shown below[9]. The third momentum vector is the one that joins the head of $\vec{P}_2$ to the foot of $\vec{P}_1$– thus ensuring that the sum of all three momenta is equal to zero[10]. This is also shown below.

[9] Remember: all of the information contained in a vector is made up of its length and angle it points in. The length represents the magnitude of the quantity (e.g. if it's a velocity vector, the length represents the speed) and the angle represents the angle that quantity acts in (e.g if it's a velocity vector, the angle represents the angle in which something is moving). The position of the vector on the page means nothing, so we are free to move the vectors around as we see fit in order to add them head-to-tail, just as we have translated $\vec{P}_2$.

[10] Remember: the total sum of all three vectors is given by the vector that joins the foot of $\vec{P}_1$ to the head of $\vec{P}_3$. In this case, since the foot of $\vec{P}_1$ and the head of $\vec{P}_3$ are in the same place, the length of the vector joining them is zero– in other words, the sum $\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$.

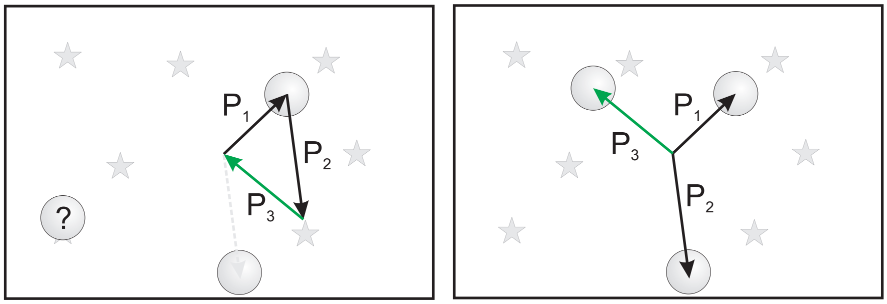
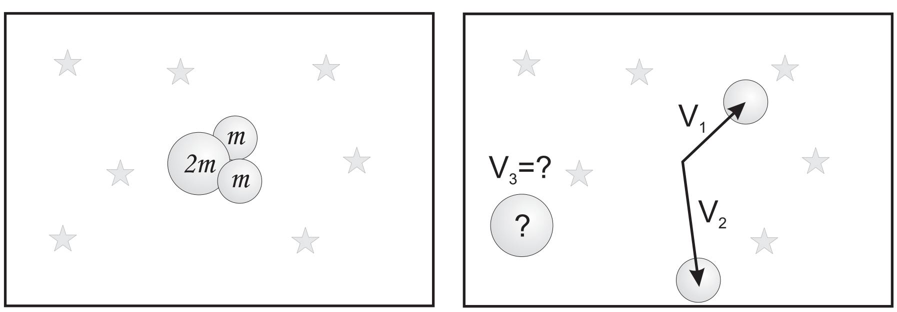


Figure 6: Question 20: LEFT: The three momentum vectors add to zero. RIGHT: The three momentum vectors translated back to the origin.

Note that the locations of the different probes being at the end of their corresponding momenta vectors generally doesn't mean anything physical– in general, it doesn't represent where their relative positions would be as momentum and velocity are not the same thing. In this special case, because all three probes have the same mass, their momentum vectors will also have the same relative lengths as their velocity vectors. See Problem 21 for an example of when the masses are not the same!

21. This is a variation on Problem 20. Three space probes are stuck together and are sitting at rest as shown in Figure 4(a). Two of the probes are identical and have a mass of m, while the third is twice as heavy, having a mass of $M = 2m$. Just as in Problem 20, a small explosive charge sets them moving away from one another, as shown in Figure 4(b). The velocity vectors of the first and second space probes are shown (V_1 and V_2, respectively). Graphically determine what the velocity vector of the third one (V_3) is in this case. Do not calculate anything– draw on Figure 4(b) what the length and direction of the V_3 vector should be, showing how you derived it. Separately, explain your reasoning in words. Note that the location of the third space probe is not correct. *Hint: Determine the different momenta of the three probes and then determine what their relative velocities should be by scaling the momenta by the different masses, since momentum = mass × velocity.* **Use Figure 5(b) if you would like a larger diagram to draw on.**



(a) The probes together

(b) The probes separate

Figure 7: Question 21: Three probes stuck together, initially at rest (a) and after a small explosive charge sends them moving away from each other (b). One probe has twice the mass of the others.

Answer: As in Problem 20, we know that the total momenta[11] of the system will be zero after the explosion, as it was zero before the explosion. The first step in solving this problem is to figure

[11] the vector sum of the three probes' momentum vectors.

out what the momenta of each of the probes is after the explosion. Since the momentum of something is defined as being the product of its mass times its velocity (not speed), we know that the momenta vectors will point in the same direction as the velocity vectors, but will have their lengths scaled by the mass of the probes (the length of a momentum vector is given by the speed of an object multiplied by its mass).

So, the momentum of the first probe, $\vec{P}_1$, is determined by multiplying its velocity, $\vec{V}_1$, by its mass, m_1. $\vec{P}_1$ will then point in the same direction as $\vec{V}_1$, but its length will be scaled by the factor m_1. Similarly, $\vec{P}_2$ points in the same direction as $\vec{V}_2$ and is scaled by the factor m_2. Now, since $m_1 = m_2$, that means that the scale factors we apply to $\vec{V}_1$ and $\vec{V}_2$ to get $\vec{P}_1$ and $\vec{P}_2$ are the same– which really means we're free to use the vectors $\vec{V}_1$ and $\vec{V}_2$ as they're drawn to also represent $\vec{P}_1$ and $\vec{P}_2$[12]. This is shown below:

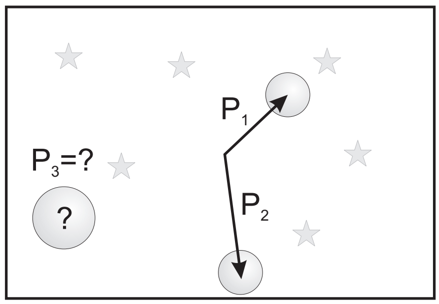
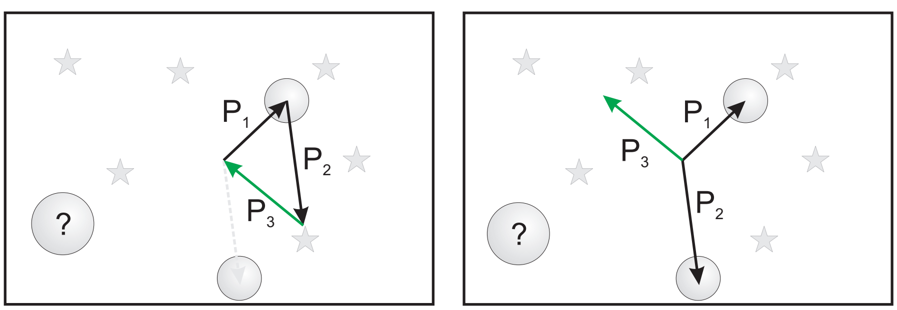


Figure 8: Question 21: The velocities of the probes have all been scaled by the factor m to change them to momenta. At the same time, the scale of the paper has been scaled *down* by the same factor of m to allow for the convenience of using the drawing of the velocities to represent the momenta.

The idea from here on out is to find $\vec{P}_3$ and then re-scale everything back into velocity vectors by dividing the probes' momenta vectors by the probes' masses.

Now to find $\vec{P}_3$... Well the way to do this is the same as in Problem 20, and the solution is shown below:



(a) The momenta translated for head-to-tail addition.

(b) The momenta translated back for clarity.

Figure 9: Question 21: The three probes' momenta.

[12]The only thing that matters is that we keep the *relative* lengths of all the momentum vectors the same. Because $\vec{P}_1$ and $\vec{P}_2$ are determined by scaling their velocities by the same quantity ($m_1 = m_2 = m$), then their velocity vectors will have the same *relative* lengths as their momentum vectors. We can also think of this in another way: the scale we're using to represent the momentum vectors on the paper (e.g. 1 cm=1 kg m/s) was scaled *down* by the same amount that the momentum vectors were scaled *up*. It doesn't matter that we are now using different scales for the momentum and velocity vectors because momentum vectors and velocity vectors are completely different things; we never can add them together or anything. So if they exist on the same piece of paper, but have different scales, that's OK.

Now we know the momenta of the different probes: $\vec{P}_1$, $\vec{P}_2$, and $\vec{P}_3$. As discussed above, it's time to find the different velocities: we have to scale the momenta by their masses. So, we divide $\vec{P}_1$ and $\vec{P}_2$ by m[13]. As we have scaled everything between momenta and velocities by this factor, on paper this means that the lengths of $\vec{P}_1$ and $\vec{P}_2$ will be the same as the lengths of $\vec{V}_1$ and $\vec{V}_2$ (and they had better!). However, since the third probe as a mass that is twice that of either the first or second probe, the length of its velocity vector, $\vec{V}_3$, will be one-half the length of its momentum vector, $\vec{P}_1$. This is shown below:

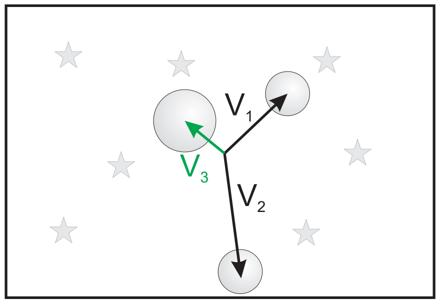


Figure 10: Question 21: The momenta of all the probes have been re-scaled (by dividing by their masses) to find their velocities.

Now, if the third probe weighed *four* times as much as either of the other two, it would still have the same momentum (i.e $\vec{P}_3$ would still have the same length and angle), but its velocity vector would only be 1/2 as long as the one shown above (i.e. it would have half the speed as shown above). And if the third probe had the same mass as either of the other two probes, then it would again have the same momentum (i.e $\vec{P}_3$ would still have the same length and angle), but the length of its velocity vector (i.e. speed) would be twice as large as shown above. *Note that in the figure above, which depicts the relative velocities of the three probes, the location of the probes as drawn does represent their relative locations after some amount of time.*

Hopefully you see how powerful the concept of *momentum conservation* is, now. Think of all the details in the problems in the problem set that you didn't need to know to understand what would happen: You didn't need to know the detailed locations of the explosive charges in Problems 20 and 21, etc. In real life, it is often easier to measure the momenta of things after a collision or explosion than it is to deal with all of the details of all the forces that go on during a collision or explosion. This has been used by physicists to great effect in particle accelerators: it has allowed them to find subatomic particles that they otherwise would have never seen!

[13] Remember: $m_1 = m_2$.