Physics 100: **Solutions** to Homework Assignment #4

Was Due on Friday, February 23^{rd} at the Beginning of Class

Section 1. **Warm-up! Fill-in-the-Blanks (1 pt each)**

1. The number of forces required for an interaction is <u>two</u>.
2. Newton's Third Law states that whenever one object exerts a force on a second object, the second object exerts a force on the first object with <u>equal</u> magnitude but in the <u>opposite</u> direction.
3. Three examples of vectors include <u>velocity</u>, <u>acceleration</u>, and <u>force</u>.
4. Three examples of scalar quantities include <u>speed</u>, <u>temperature</u>, <u>mass</u>.
5. Acceleration always points in the same direction as <u>Net Force</u>.
6. When acceleration points in the same direction as the velocity of a car, the car's speed <u>increases</u>. When acceleration points in the opposite direction as the velocity, the car's speed <u>decreases</u>. When the acceleration points at a right angle to the velocity the car's speed <u>doesn't change</u>, but the car does <u>change direction</u>. If the acceleration points at an angle (not 0°or 90°) relative to the velocity, then both the car's <u>speed</u> and <u>direction</u> change.
7. When hitting a nail with a hammer, if the action is defined as the hammer hitting the nail, then the reaction is the <u>nail</u> hitting the <u>hammer</u>.
8. A hammock stretched tightly between a pair of trees is <u>more</u> likely to break than one that sags more when someone sits in it.
9. Two vectors that add together to equal zero point in <u>opposing</u> directions and have <u>equal</u> magnitudes.

Section 2. **Short Answer Questions (2 pts. each)**

10. If the forces that act on a cannonball and the recoiling cannon from which it is fired are equal in magnitude, why do the cannonball and cannon have very different accelerations?

Answer: It is because they have very different masses. Newton's Second Law tells us that the net force on an object is equal to the product of its mass and acceleration. Since the force on the cannon and cannonball are the same, we can write:

$$mass_{cannon} \times accel_{cannon} = Force = mass_{cannonball} \times accel_{canonball}$$

We can see that if the mass of the cannon is much larger than the mass of the cannonball, its acceleration must be much smaller. This may be easier to see if we divide each side of the equation by the mass of the cannon:

$$accel_{cannon} = \frac{mass_{cannonball}}{mass_{cannon}} \times accel_{canonball}$$

Saying that the mass of the cannon ball is much smaller than the mass of the cannon is the same as saying that $\frac{mass_{cannonball}}{mass_{cannon}}$ is much smaller than one. For example, take a mass of 10 kg for the cannonball and 1000 kg for the mass of the cannon. The ratio is 0.01, or 1%. This means that the acceleration of the cannon is 1% of the acceleration of the cannonball. So if a cannon ball accelerates at 10 g's (100 m/s^2), then the cannon will accelerate at only 1 m/s^2.

11. When you walk along the floor, you move forward because something provides a force. (A) What is pushing you forward, and (B) what force is it using?

    Answer: (A) The floor is pushing you forward by (B) using the force of friction.

12. When you walk along the Earth, you don't change the Earth's rotation very much. Why is it that people participating in log-rolling competitions change the rotation of the log by an awful lot?

    Answer: In both cases, you are providing the same amount of force on what you're walking on as that something does to you, in accordance with Newton's Third Law. In the case of the Earth, because it is so massive, the force you apply is too small to get it to accelerate (change its motion). In the case of the log, it is not so massive, so the force you exert by walking will accelerate it a lot (changing its motion).

13. Within a book on a table, there are billions of forces pushing and pulling on all the molecules. (A) Why is it that these forces never, by chance, add up to a net force in one direction, causing the book to accelerate "spontaneously" across the table? (B) How does your answer refute Aristotle's idea of *Natural Motion*?

    Answer: (A) If we define as our system the whole book, then every one of these forces acting on the molecules inside the book is canceled by an equal and opposite reaction, so that the sum total of all the forces is always zero. Consider just a pair of molecules acting on one-another and imagine that the force they interact with is very similar to the situation where they're tied together by bungee a cord. If one molecule pulls on the other molecule in order to move it in forward, the other molecule will pull on it and move it backward. The net result is that the two molecules came closer together, but as a system, they didn't move anywhere. (B) Taking Newton's Three Laws as truth, it pretty much debunks Aristotle's idea of natural motion because it shows how objects cannot spontaneously accelerate (e.g. come to a stop) without an outside force affecting them, such as friction.

14. Why does someone climbing on a rope pull down on the rope in order to move up?

    Answer: The climber pulls down on the rope so that the rope will push up on her, causing her to move upwards if the force is larger than her weight.

15. A farmer urges his horse to pull a wagon. The horse refuses, saying that to try would be futile for it would violate Newton's Third Law for if the horse were to pull on the wagon, the wagon would pull back with an opposite and equal force, making the net force between them be zero and thus leading to a zero acceleration. After deciding that he should really cash in on his talking horse on the talk shows, what does the farmer say to refute the horse's argument and convince it to pull the wagon? (Ignore the friction of the cart wheels on the road).

    Answer: The horse is full of it. What the horse is proposing is something like what's depicted in Figure 1, and is assuming that the force pair cancels out so that nothing will move.

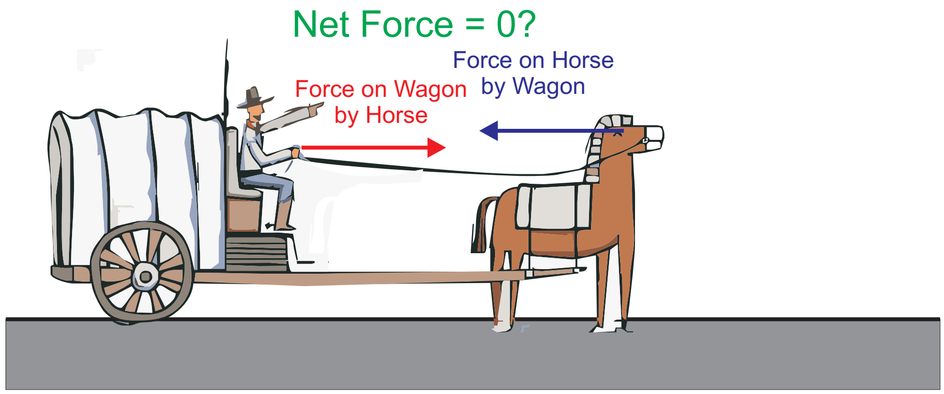

Figure 1: All of the forces, according to the Horse, for Problem 15

However, the horse is forgetting something rather important, and that's the force pair of the horse and road. This is shown in Figure 2.

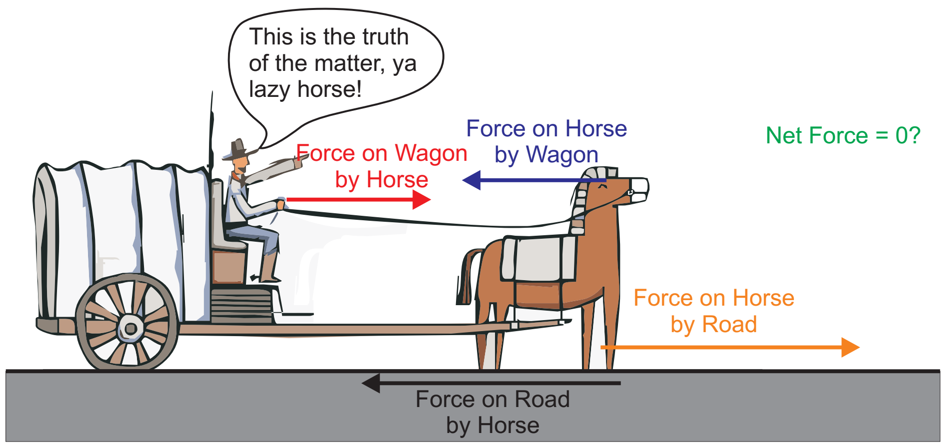

Figure 2: All of the forces, including the Horse/Road force pair for Problem 15

It's this extra *force on the horse by the road* that pushes the system forward. This diagram is rather cluttered and is the kind of thing that leads the horse to making such a mistake. It is best to define a system. Once the system is defined, then only the forces acting on that system should be considered. For example, if the horse had only considered the system to be *Wagon+Horse*, as shown in Figure 3, it would have been obvious to it that there is a net force forward which is responsible for moving the cart (the *force on the horse by the road*) *via* friction).

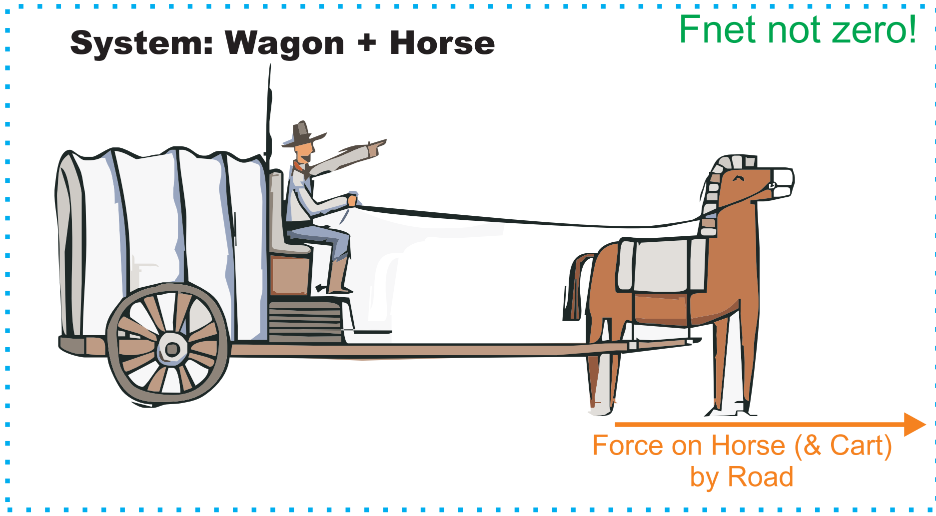

Figure 3: Using the Cart+Horse as a system, it is easy to see that there is a net force. The forces of the horse and cart on each other are simply internal forces and don't appear, any more than you would draw the force pair of every atom in a raindrop when talking about it falling through the air. Problem 15

To better understand the assignment of these systems and where all of the forces should go, let's look at the wagon and horse alone, as in Figure 4.Only the external forces acting on the systems are drawn. In the case of the wagon system, one need only consider the *force on the wagon by the horse* (we're ignoring friction between the road etc.). It is clear that there is a net force accelerating the wagon to the right.

For the horse, it is a bit more hazy, as there are two opposing forces acting: the *force on the horse by the wagon* and the *force on the horse by the road*. We know, however, that since the horse and wagon move forward together, that the *force on the horse by the road* must be greater than the *force on the horse by the wagon* because (1) we see that the entire system is moving forward and (2) we see that the wagon is moving forward.

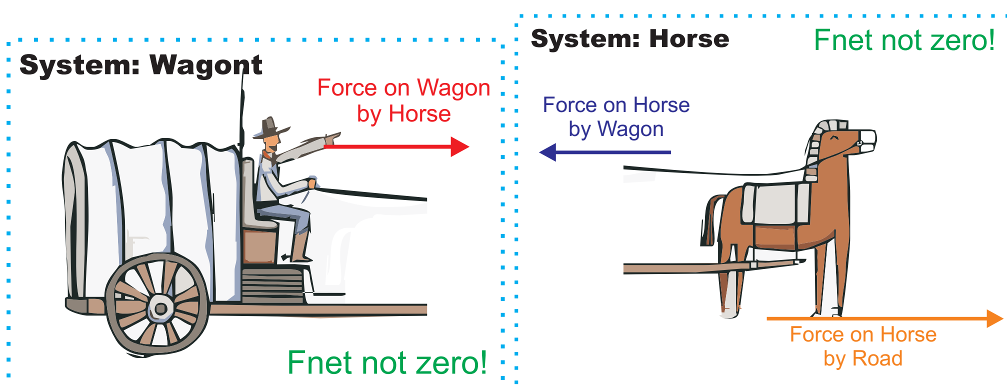

Figure 4: Defining the cart and horse systems separately. See Problem 15

16. Two people, named Left and Right, of *equal* mass attempt a tug-of-war with a 12-m rope while standing on frictionless ice. When they pull on the rope, each of them slides toward the other. (A) How do their accelerations compare, and (B) how far does each person slide before they meet? Now consider the case where Left weighs *twice* as much as Right. (C) How do their accelerations compare in this case?

Answer: Let us consider the system of each person, Left and Right, separately. The vertical forces acting on Left are the force of gravity and the support force. They cancel each other out so there is no net force component in the vertical direction. In the horizontal direction, there is the *force on Left by Rights* and nothing else (we're ignoring friction). This force will cause Left to accelerate towards Right, according to Newton's Second Law:

$$a_{Left} = \frac{F_{on\ Left\ by\ Right}}{m_{Left}}$$

where a_{Left} is Left's acceleration, $F_{on\ Left\ by\ Right}$ is the *force acting on Left caused by Right*, and m_{Left} is Left's mass.

Now, considering Right: the vertical forces are treated the same as in the case of Left, so we are left to conclude that there is also no vertical force for Right. The horizontal force is treated in a similar way, too, and we conclude that for Right:

$$a_{Right} = \frac{F_{on\ Right\ by\ Left}}{m_{Right}}$$

where a_{Right} is Right's acceleration, $F_{on\ Right\ by\ Let}$ is the *force acting on Right caused by Left*, and m_{Right} is Right's mass.

$F_{on\ Left\ by\ Right}$ and $F_{on\ Right\ by\ Left}$ are proper force pairs, so Newton's Third Law dictates that they must be of equal magnitude. Since m_{Right} and m_{Left} are also equal, we conclude that a_{Right} and a_{Left} are the same.

(B) Since Left and Right have the exact same acceleration towards each other, and because they have the same time of travel, the following equation[1] tells us that they will travel the same distance:

$$Distance\ traveled = \frac{1}{2} \times acceleration \times (time)^2$$

Since they travel the same distance, and because the total distance covered by them combined is 12 m, we know that they each travel 6 m towards one another.

(C) Now Left weighs twice as much as Right:

$$m_{Left} = 2m_{Right}$$

Everything is treated the same as in (A), and we find that

$$a_{Left} = \frac{F_{on\ Left\ by\ Right}}{m_{Left}}$$

$$a_{Right} = \frac{F_{on\ Right\ by\ Left}}{m_{Right}}$$

Again, $F_{on\ Left\ by\ Right} = F_{on\ Left\ by\ Right}$ since they are proper force pairs of an interaction. However, since Left's mass is twice that of Right's mass, we see that Left's acceleration is 1/2 that of Right's:

$$a_{Left} = \frac{F_{on\ Left\ by\ Right}}{m_{Left}}$$

$$a_{Left} = \frac{F_{on\ Left\ by\ Right}}{2m_{Right}}$$

$$a_{Left} = \frac{1}{2} \times \frac{F_{on\ Left\ by\ Right}}{m_{Right}}$$

$$a_{Left} = \frac{1}{2} \times \frac{F_{on\ Right\ by\ Left}}{m_{Left}}$$

$$a_{Left} = \frac{1}{2}\ a_{Right}$$

[1] We can use this equation because they started with a zero velocity!

17. Your little brother wants to hang up a Halloween decoration from the ceiling. He is using weak string and only has one piece. He is trying to decide which way to hang the witch and has two choices (A) and (B), as shown in below. He asks you for help. If he breaks the string and can't hang up his witch decoration, he'll annoy you with non-stop sobbing and whining. Which witch hanging do you recommend, and why?

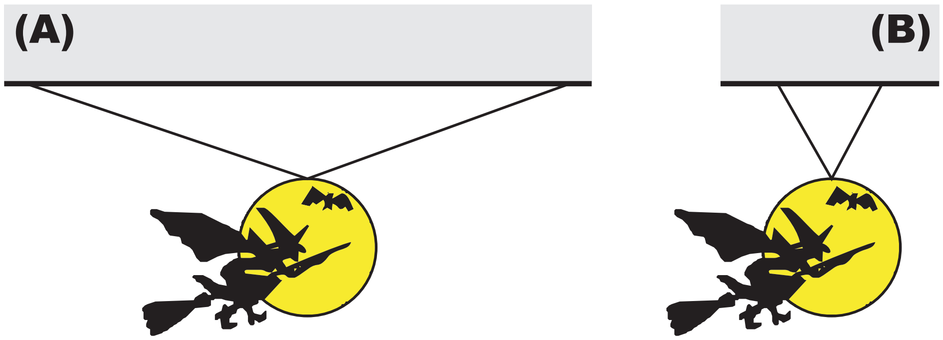

Figure 5: Keep your little brother from sobbing! Which witch will work? See Problem 17

Answer: Choosing the correct witch-hanging configuration amounts to finding the one that puts the least tension in the string. Starting off, we draw the free-body diagram of the hanging witch as shown in Figure 6.

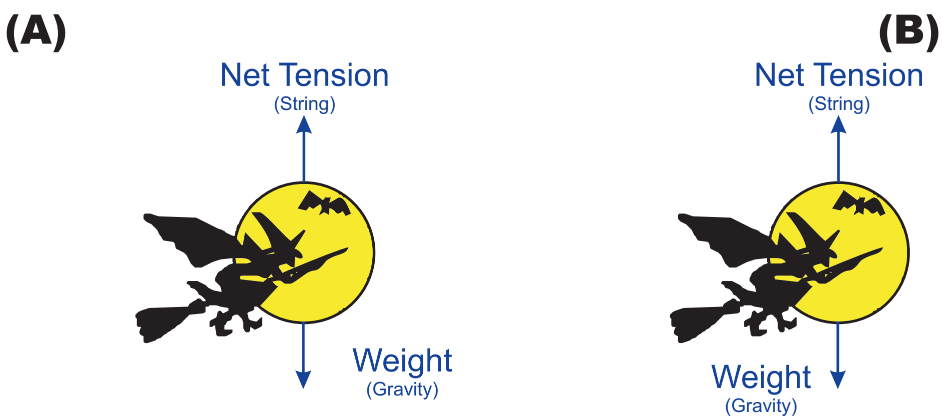

Figure 6: The free-body diagram for the Witch. See Problem 17

The witch is affected by *gravity* (i.e. it's weight), which pulls it down, and a *net tension force* (due to the tension in the string) pulling it up. Because the witch, if it's hung properly, will be in equilibrium (it won't be accelerating in any direction– it will hang still), then the *weight force* and the *net tension force* have to cancel each other out to ensure that the net force is equal to zero. Since the witch weighs the same regardless of how it's hung, we see that the net tension force for both cases must also be the same.

The net tension force is due to the tension in the string pulling in two different directions: one up and towards the left, and the other up and towards the right– each acting along the direction of the string, as shown in Figure 7.

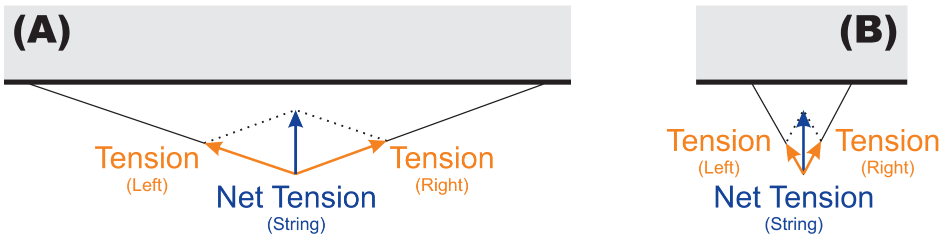

Figure 7: Resolving the tension components from the net tension. See Problem 17

We can determine the magnitudes of these two components for each of the different choices by using the parallelogram rule. As the name implies, this means drawing a parallelogram, with two sides defined by the string, and two more sides drawn in by you that are parallel to the two string sides and intersect at the end of the net tension vector. When we do this, we see that the tension in choice (A) is much higher than choice (B). Qualitatively, we can understand this by seeing that the more level the string, the more the left and right sides of the string are pulling against each other in the horizontal direction, and only a small fraction of the total tension goes into pulling up vertically. The larger the kink, the more they pull up on the witch compared to fighting each other in the horizontal direction. Basically, if you want to get as much useful force out of your rope, you want to keep the two bends going in the same direction so they don't use their tension fighting each other. The ideal case would be one not shown here: one where the string bends over on itself and hangs vertically. In this case, all of the tension in the string will go into lifting the witch. If you think about it, this is why tennis court nets are held up with steel ropes that have a huge tension in them.

18. (A) Can two vectors with unequal magnitudes ever add to zero? (B) Can three unequal vectors add to zero? Defend your answers.

Answer: (A) No. The only way *two* vectors could cancel each other out is if they acted in opposing directions. If the two vectors have different magnitudes, then there is always a bit left over, in the direction of the larger vector, as illustrated.

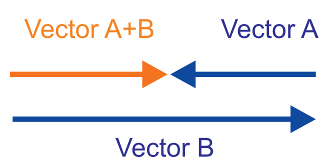

Figure 8: Two vectors of unequal magnitudes could never add to zero (cancel each other out). See Problem 18

(B) Yes. Three vectors with unequal magnitudes (or equal, for that matter) *can* sum to zero (cancel each other out). An example of this is shown in Figure 9. When vectors sum up to zero (cancel each other out), they can be arranged in a chain, "from head to tail," where the start and finish point are the same. This is easy to imagine when you consider displacement vectors. If you start at the tail of $\vec{A}$[2], move along $\vec{A}$, then B$\vec{B}$, and finally $\vec{C}$,, you end up where you started. The net result would have been the same is you had just stayed put (a resultant of magnitude zero). So the problem to show that three vectors can cancel each other out is the same as showing that a triangle can be made out of them. This happens whenever the magnitude of the smallest two vectors, when added together, is greater than or equal to the the magnitude of the largest vector. If the sum of the two smallest magnitudes exactly equals the largest magnitude, then the two smallest vectors will point in the same direction, which is opposite and equal to the third, largest

[2]Remember: the arrow over the A means that A is a vector!

vector. If their sum of the smallest vectors' magnitudes exceeds the magnitude of the largest, then they form a triangle.

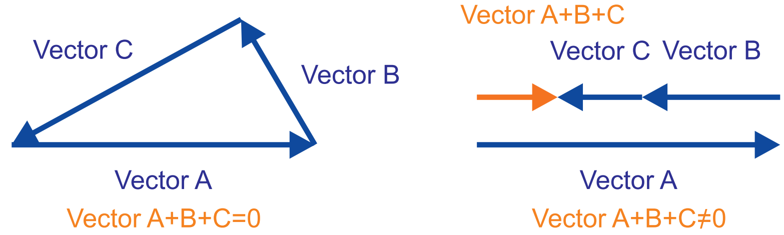

Figure 9: Three vectors can add cancel each other out if sum of the magnitudes of the two smallest vectors is equal to or greater than the magnitude of the third. See Problem 18

19. A duck is swimming across a river. The duck can swim at a maximum speed of 2 m/s in still water. If the duck is swimming as hard as possible straight across a river that's running at 4 m/s, what is the duck's overall speed, relative to the goose sitting on the river bank?

Answer: Since the duck is swimming at it's maximum speed across the river, it has a cross-river velocity component of 2 m/s. The river sweeps the duck with it, giving the duck a down-river velocity component of 4 m/s. The speed of the duck is given by the magnitude of its total velocity[3]. In this case, the cross-river and down-river directions are perpendicular to one-another, so the velocity components add like the legs of a right triangle, so that their resultant (total velocity) has the magnitude (speed) of:

$$speed = \sqrt{(2\ m/s)^2 + (4\ m/s)^2}$$
$$speed = \sqrt{4\ m^2/s^2 + 16\ m^2/s^2}$$
$$speed = \sqrt{20\ m^2/s^2}$$
$$speed = \sqrt{20}\ m/s$$

20. Consider the diagram showing a cylinder at rest against a large square on a hill. (A) Identify all the forces that act on the cylinder. (B) Show how all of the forces acting on the cylinder cancel each other out by drawing the forces to scale. Hint: decompose the forces properly, starting with the weight.

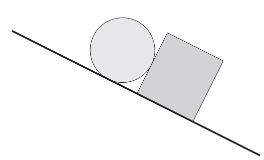

Figure 10: A cylinder is at rest against a large block on an incline. See Problem 20

Answer: There are three forces acting on the cylinder: the force of gravity ($\vec{F}_g$[4]), the support force (normal force) from the hill ($\vec{F}_{N,hill}$), and the support force (normal force) from the block ($\vec{F}_{N,block}$). $\vec{F}_g$ points straight down, while the normal forces point normal (i.e. 90 degrees) to the surface that causes them: ($\vec{F}_{N,hill}$) points perpendicular to the hillside and ($\vec{F}_{N,block}$) points perpendicular to the block, which, in this case, happens to be parallel to the hillside. This is

[3] Its total velocity is given by the vector addition of its cross-river velocity and it's down-stream velocity.
[4] Remember: the arrow over the F means we're talking about a vector!

shown in the Figure 11.Note that at this point, we don't know how to draw the relative magnitudes (i.e. lengths) of the these three vectors, only their relative directions. In order to determine their relative magnitudes, we first have to fix one of the vector's lengths and say that it represents a known quantity. I think the easiest one to choose is weight of the cylinder, $\vec{F}_g$. This is shown in Figure 11by having question marks next to $\vec{F}_{N,hill}$ and $\vec{F}_{N,block}$ but not $\vec{F}_g$. Whatever the cylinder may actually weigh, it's weight will be represented by the length (magnitude) of $\vec{F}_g$[5].

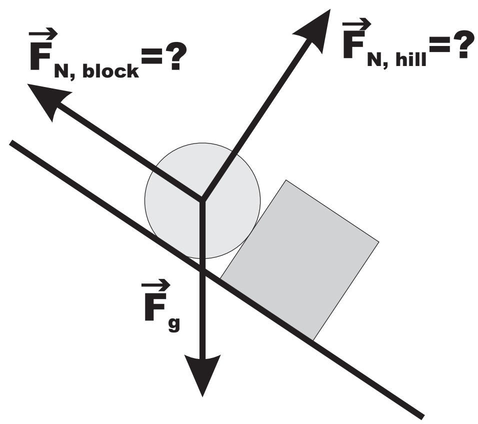

Figure 11: A cylinder is at rest against a large block on an incline with $\vec{F}_g$, $\vec{F}_{N,hill}$, and $\vec{F}_{N,block}$. At this point, only the relative directions of the three forces are known– their relative magnitudes are unknown. At this stage, we have defined the length of $\vec{F}_g$ to be the weight of the cylinder. See Problem 20

By deciding on the length of $\vec{F}_g$, it is now possible to determine the lengths of $\vec{F}_{N,hill}$ and $\vec{F}_{N,block}$. The first step is to note that $\vec{F}_{net} = 0$, which implies that

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_{N,hill} + \vec{F}_{N,block} = 0$$

From here, there are a couple of different ways to solve the problem. The first way is to graphically ensure that $\vec{F}_g$, $\vec{F}_{N,hill}$, and $\vec{F}_{N,block}$ sum to zero. We can determine the lengths of $\vec{F}_{N,hill}$ and $\vec{F}_{N,block}$ relative to that of $\vec{F}_g$. Shown below in Figure 12, are $\vec{F}_g$, $\vec{F}_{N,hill}$, and $\vec{F}_{N,block}$, drawn head-to-tail. It is obvious that as they are drawn, they do not add to a zero resultant (i.e. $\vec{F}_{net} \neq 0$).

[5]In other words, we *define* the length of $\vec{F}_g$ to represent whatever weight the cylinder may have– if it's 10 N, then the length of $\vec{F}_g$ is equivalent to 10 N. If it's 500 N, then the length represents 500 N.

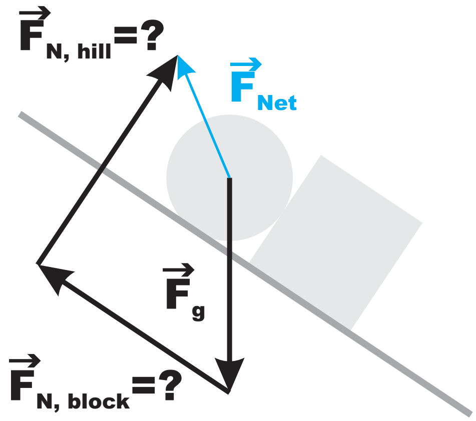

Figure 12: The final solution to Problem 20

For them to add to zero, we see that $\vec{F}_{N,block}$ has to be shorter so that $\vec{F}_{N,hill}$ will go back through the starting point (the tail of $\vec{F}_g$), as shown below in Figure 13.

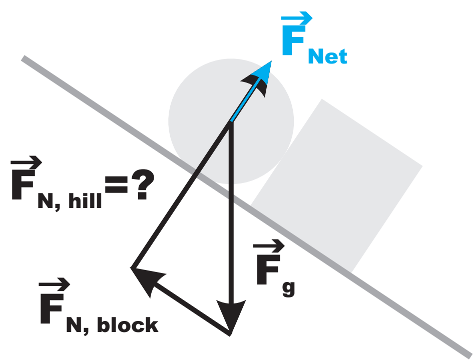

Figure 13: The final solution to Problem 20

Then all we need to do is shorten $\vec{F}_{N,hill}$ so that it will end back at the beginning (i.e. the tail of $\vec{F}_g$), as shown below in Figure 14. This combination of $\vec{F}_g$, $\vec{F}_{N,hill}$, and $\vec{F}_{N,block}$ now guarantees that $\vec{F}_{net} = 0$ and we are done.

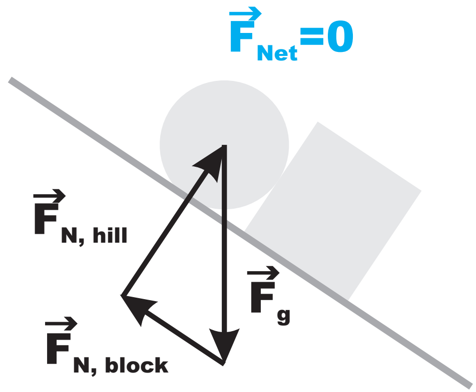

Figure 14: The final solution to Problem 20

The other way to solve this problem is by breaking down $\vec{F}_g$ into two vectors, one that is parallel to the surface of the incline (and parallel $\vec{F}_{N,block}$) and one that is perpendicular to it (and parallel to $\vec{F}_{N,hill}$). We'll call the first one $\vec{F}_{g,\parallel}$ and the second one $\vec{F}_{g,\perp}$. In vector math language:

$$\vec{F}_g = \vec{F}_{g,\parallel} + \vec{F}_{g,\perp}$$

Starting back at Figure 11and again noting that

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_{N,hill} + \vec{F}_{N,block} = 0$$

we can then express $\vec{F}_{net}$ in terms of these components and solve the problem.
We first determine what $\vec{F}_{g,\parallel}$ and $\vec{F}_{g,\perp}$ by using the same method that we used above to find the relative lengths of $\vec{F}_{N,hill}$ and $\vec{F}_{N,block}$: we know the directions that $\vec{F}_{g,\parallel}$ and $\vec{F}_{g,\perp}$ point in, and to determine their relative lengths, we have to make sure that they add up to $\vec{F}_g$.
Shown in Figure 15are the two components $\vec{F}_{g,\parallel}$ and $\vec{F}_{g,\perp}$. We see that the way they are drawn, they do indeed add to $\vec{F}_g$[6]. Note that we no longer need to explicitly consider $\vec{F}_g$, as dealing with $\vec{F}_{g,\parallel}$ and $\vec{F}_{g,\perp}$ accomplishes the same thing[7].

[6] Remember adding vectors is done either by using the parallelogram method described in the textbook, or by placing them 'head-to-tail' as described in class. If you have any questions about this, come and see me during my office hours!

[7] In fact, it would be wrong to consider $\vec{F}_g$, $\vec{F}_{g,\parallel}$, and $\vec{F}_{g,\perp}$ *all* acting on the cylinder at once, as that would effectively double the weight of the cylinder.

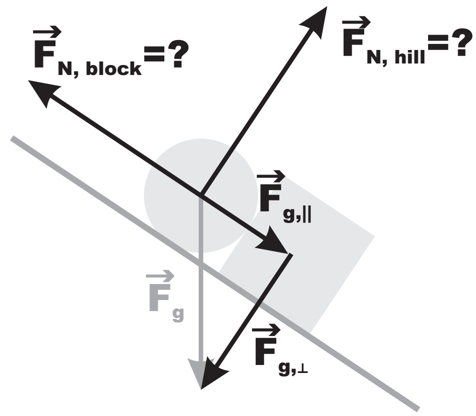

Figure 15: A cylinder is at rest against a large block on an incline with $\vec{F}_g$, $\vec{F}_{N,hill}$, and $\vec{F}_{N,block}$. $\vec{F}_g$ has been expressed as the sum of $\vec{F}_{g,\|}$ and $\vec{F}_{g,\perp}$ to make it easier to determine the magnitudes (lengths) of $\vec{F}_{N,hill}$ and $\vec{F}_{N,block}$. Refer to Problem 20

It is immediately obvious that, for $\vec{F}_{net} = 0$, $\vec{F}_{N,block}$ must be canceled by $\vec{F}_{g,\|}$ and $\vec{F}_{N,hill}$ must be canceled by $\vec{F}_{g,\perp}$, which is equivalent to saying that the lengths of $\vec{F}_{N,block}$ and $\vec{F}_{g,\|}$ must be the same, and that the lengths of $\vec{F}_{N,hill}$ and $\vec{F}_{g,\perp}$ must be the same, as shown below in Figure 16.

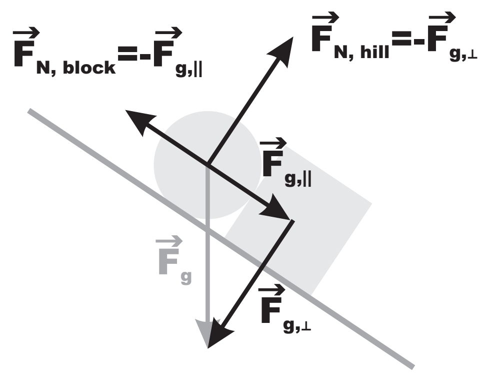

Figure 16: The final solution to Problem 20